Light Sources I

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Outline

- Introduction
- Fundamentals of Light and SR
- Overview of SR Light Source
- Characteristics of SR (1)
- Characteristics of SR (2)
- Practical Knowledge on SR

Lecture I

Lecture II

Outline

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- Characteristics of SR (2)
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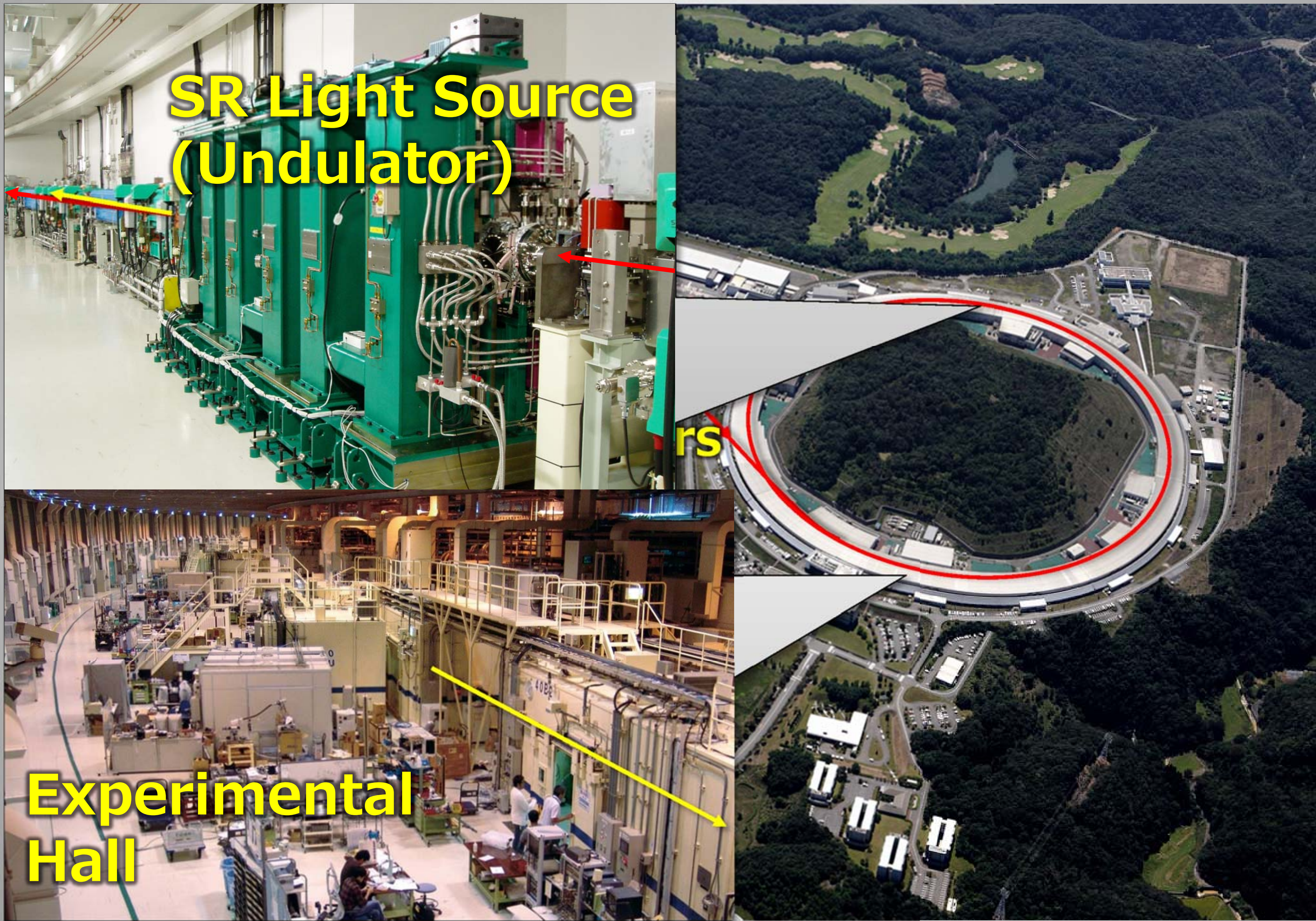
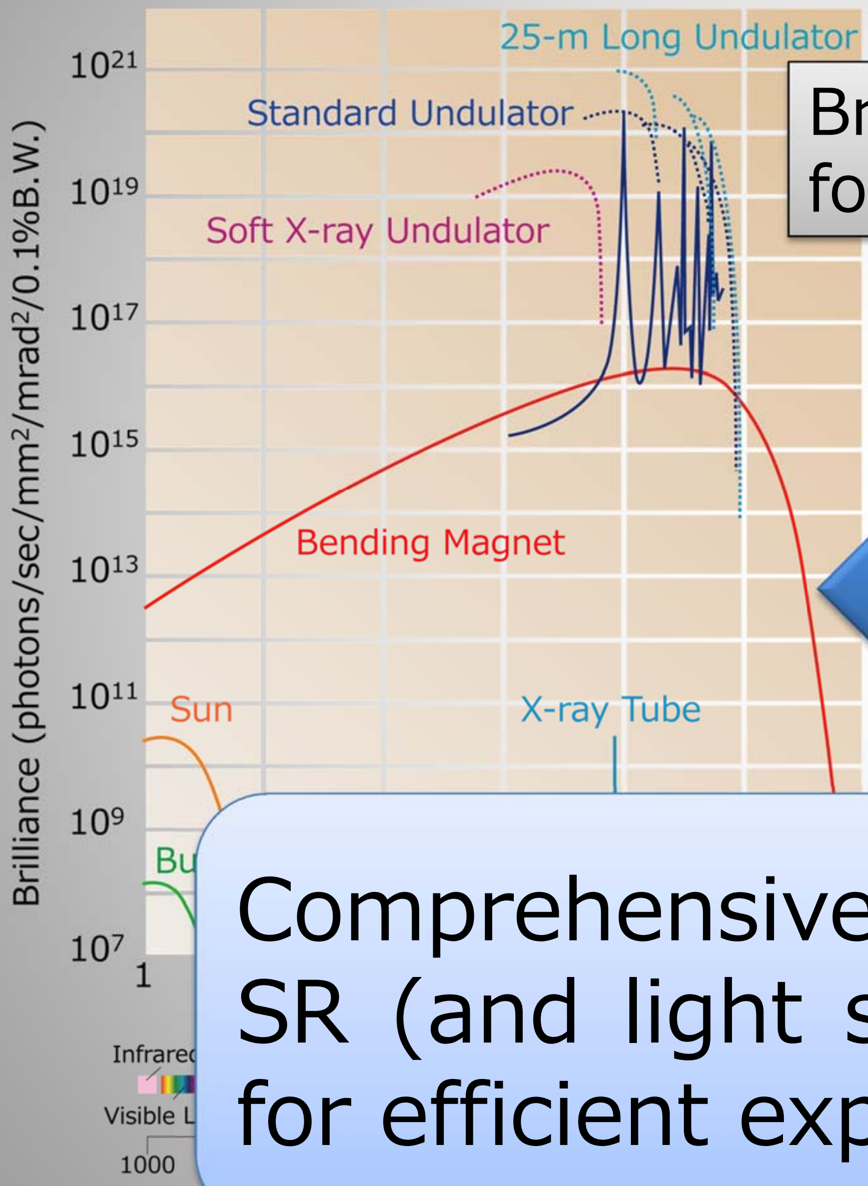
Overview of SR Facility



What's the Advantage of SR?



Brilliance vs. wavelength for various light sources

Light Source	Total Power (W)
Sun	10^{26}
X-ray Tube	10^{3}
Light Bulb	10^{2}
SR	**$10^{3} \sim 10^{4}$**

Comprehensive understanding of SR (and light source) is required for efficient experiments.

Topics in This Lecture (1)

- Fundamentals of Light and SR
  - General description of light
  - Why we need SR?
  - Physical quantity of light
  - Uncertainty of light: Fourier and diffraction limits
  - SR: Light from a moving electron
- Overview of SR Light Source
  - Types of light sources
  - Magnet configuration

Topics in This Lecture (2)

- Characteristics of SR
  - Radiation from bending magnets
  - Electron Trajectory in insertion devices
  - Radiation from insertion devices
- Practical Knowledge on SR
  - Finite emittance and energy spread
  - Heat load and photon flux
  - Evaluation of optical properties of SR
  - Definition of undulators and wigglers
  - Numerical examples

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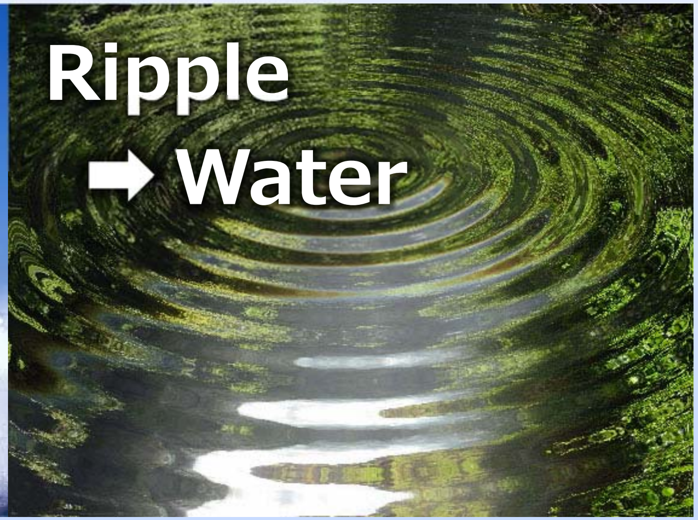
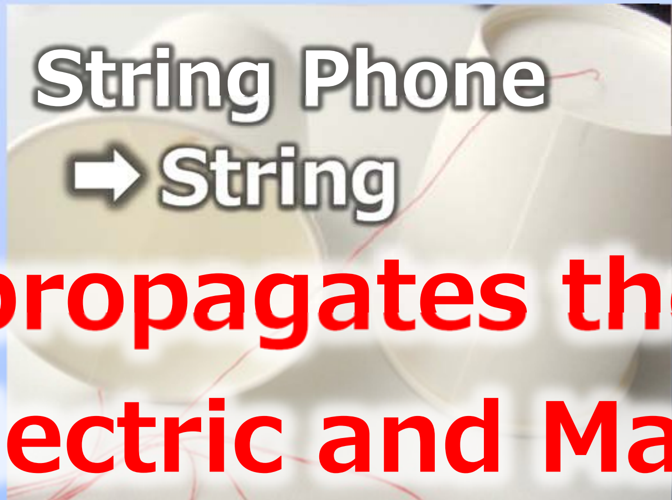
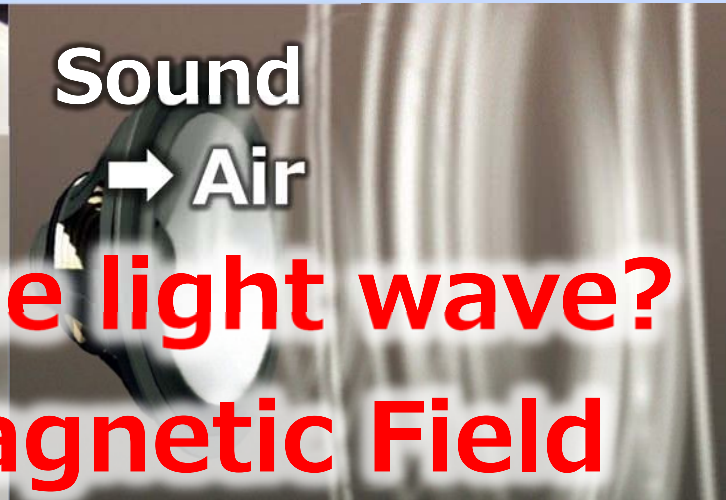
What is Light?

What is light? It is a kind of wave, but···



Various Waves

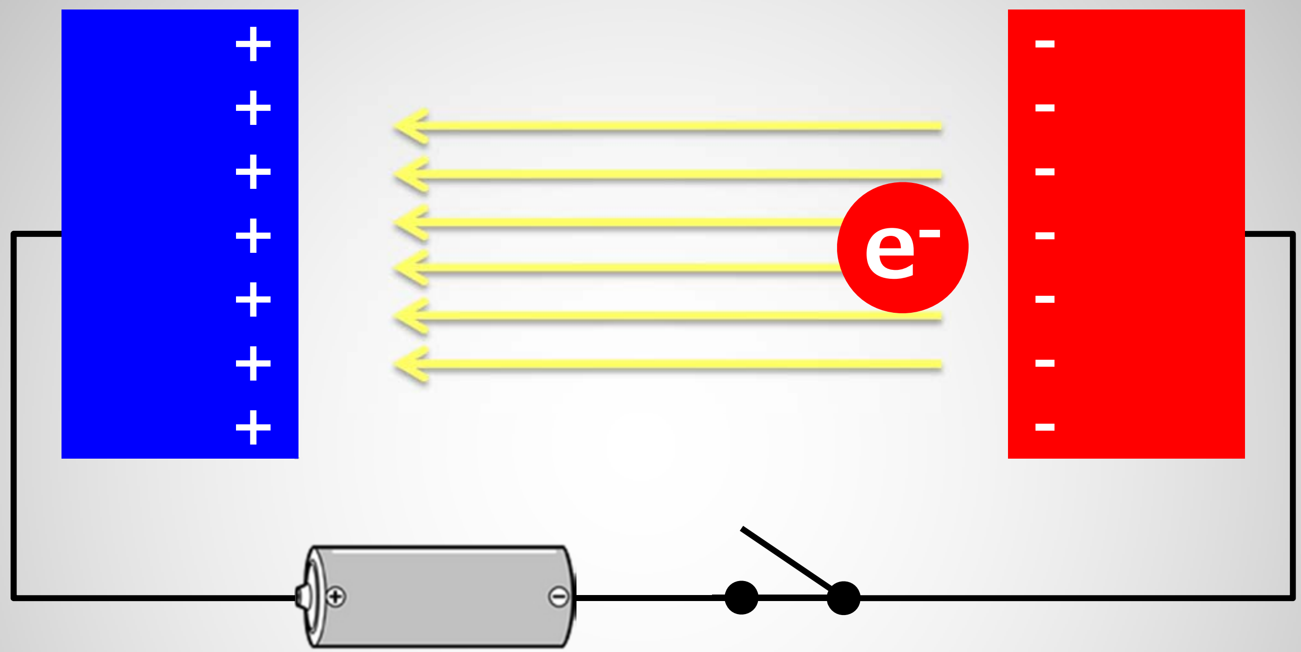




What propagates the light wave?

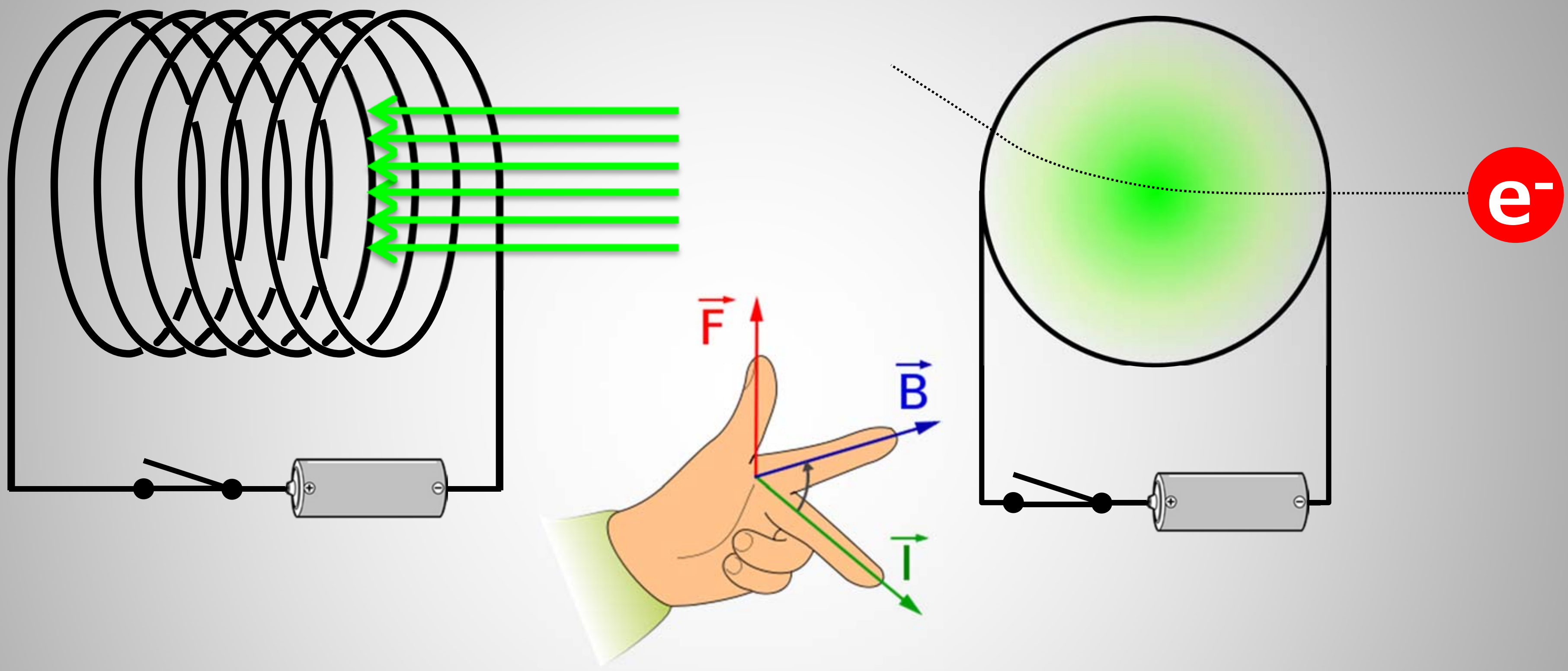
➡ Electric and Magnetic Field

Electric Field



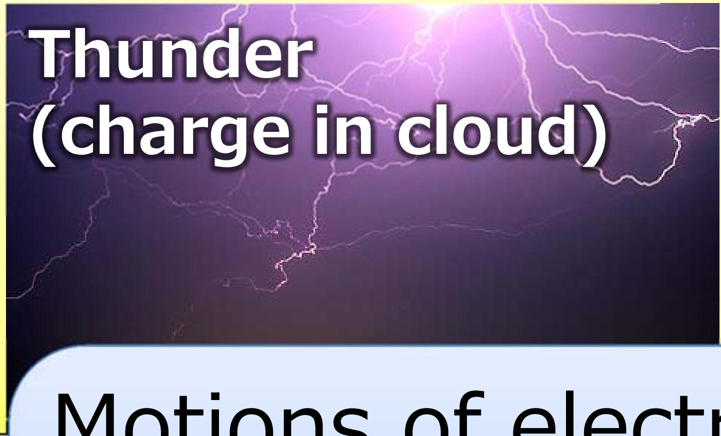
The E-field is generated by electric charges, and gives a force on a charged particle.

Magnetic Field



The M-field is generated by moving electric charges, and give a force on a moving charged particle.

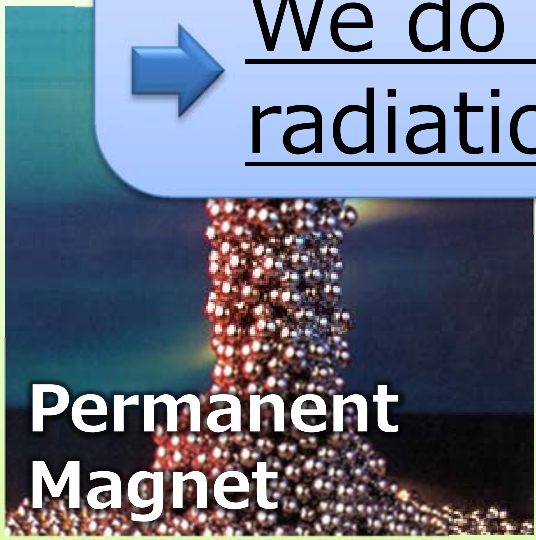
Electro- and Magnetostatic Phenomena

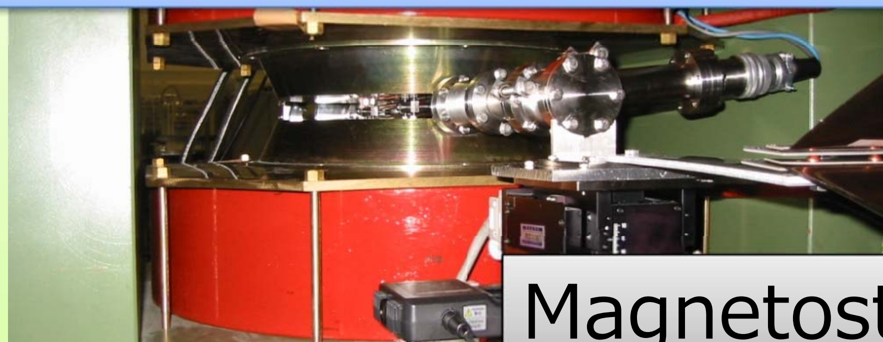


Elect

Motions of electrons (charges) are stationary or steady.

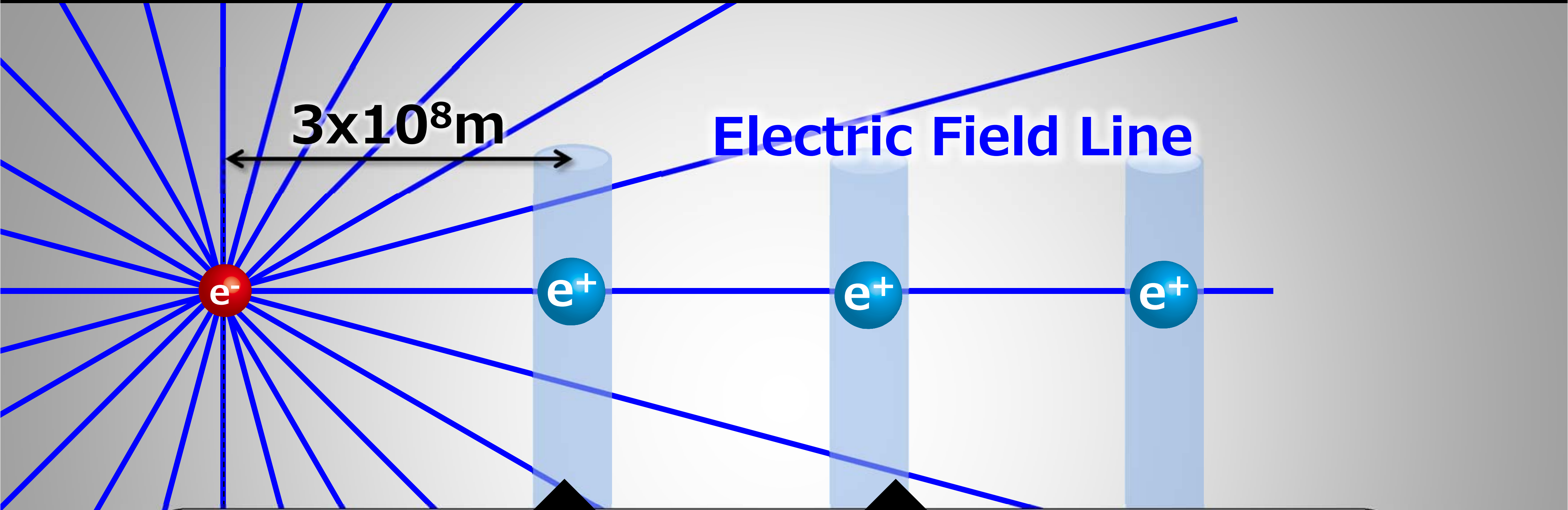
→ We do not observe any radiation in such a steady state.





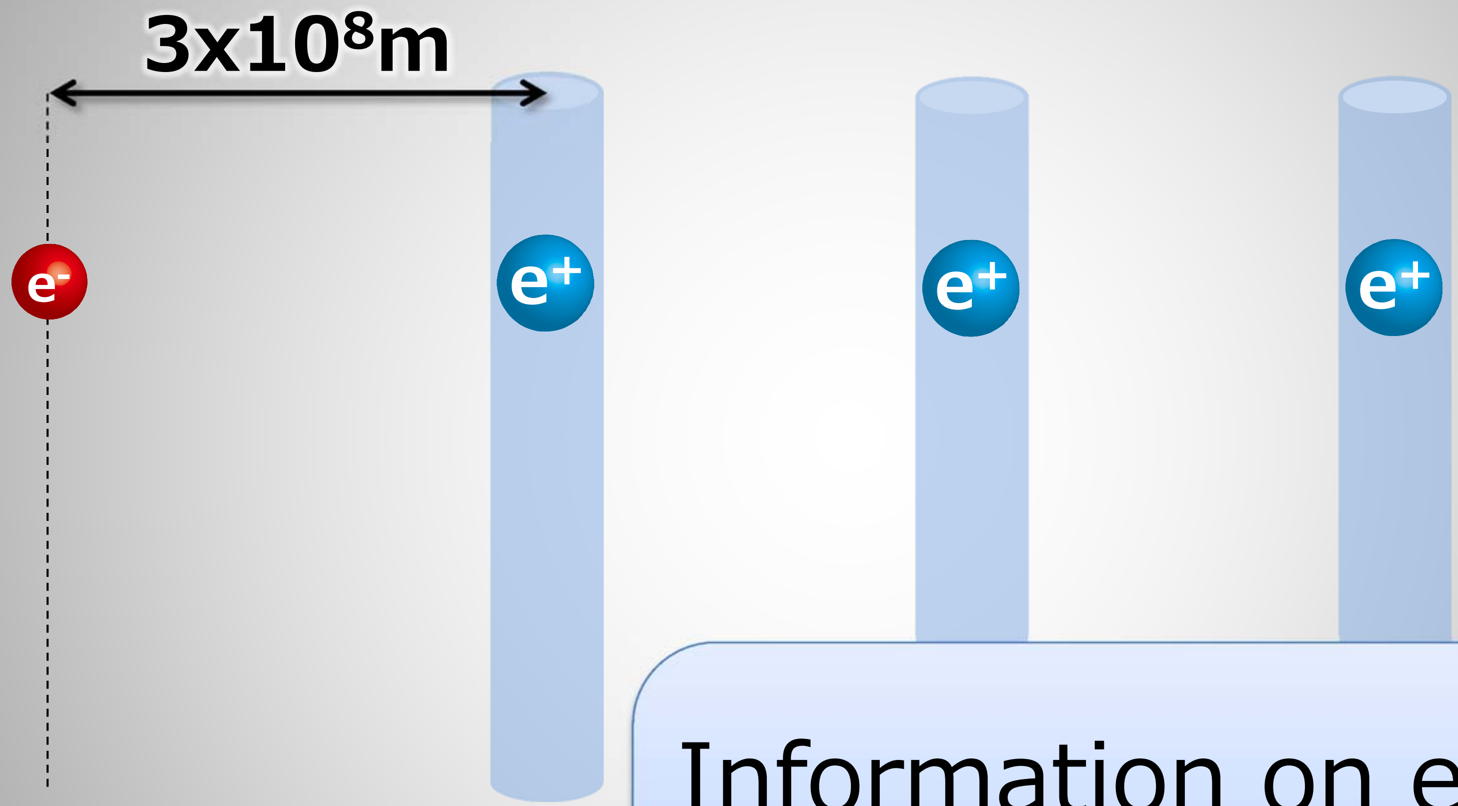
Magnetostatic

Non-Steady State: Thought Experiment



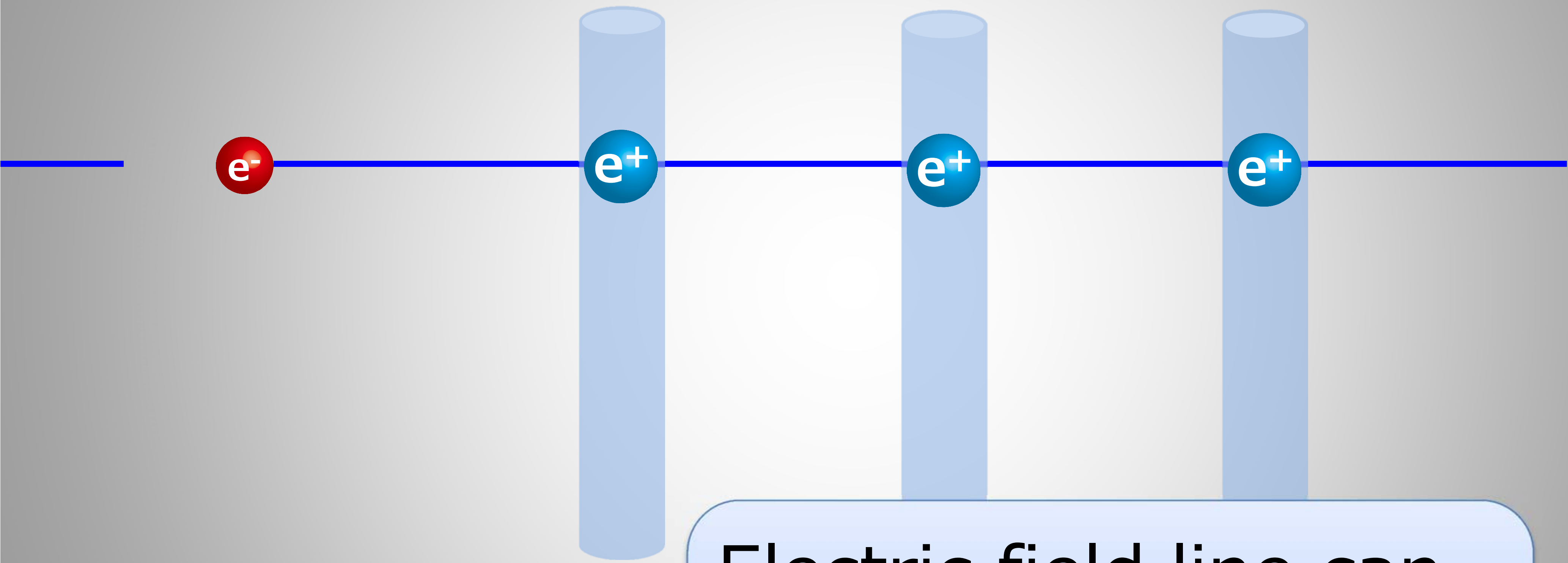
~~Does e^+ moves simultaneously? If so, the information of e^- propagates with an infinite speed!~~

Non-Steady State: Thought Experiment



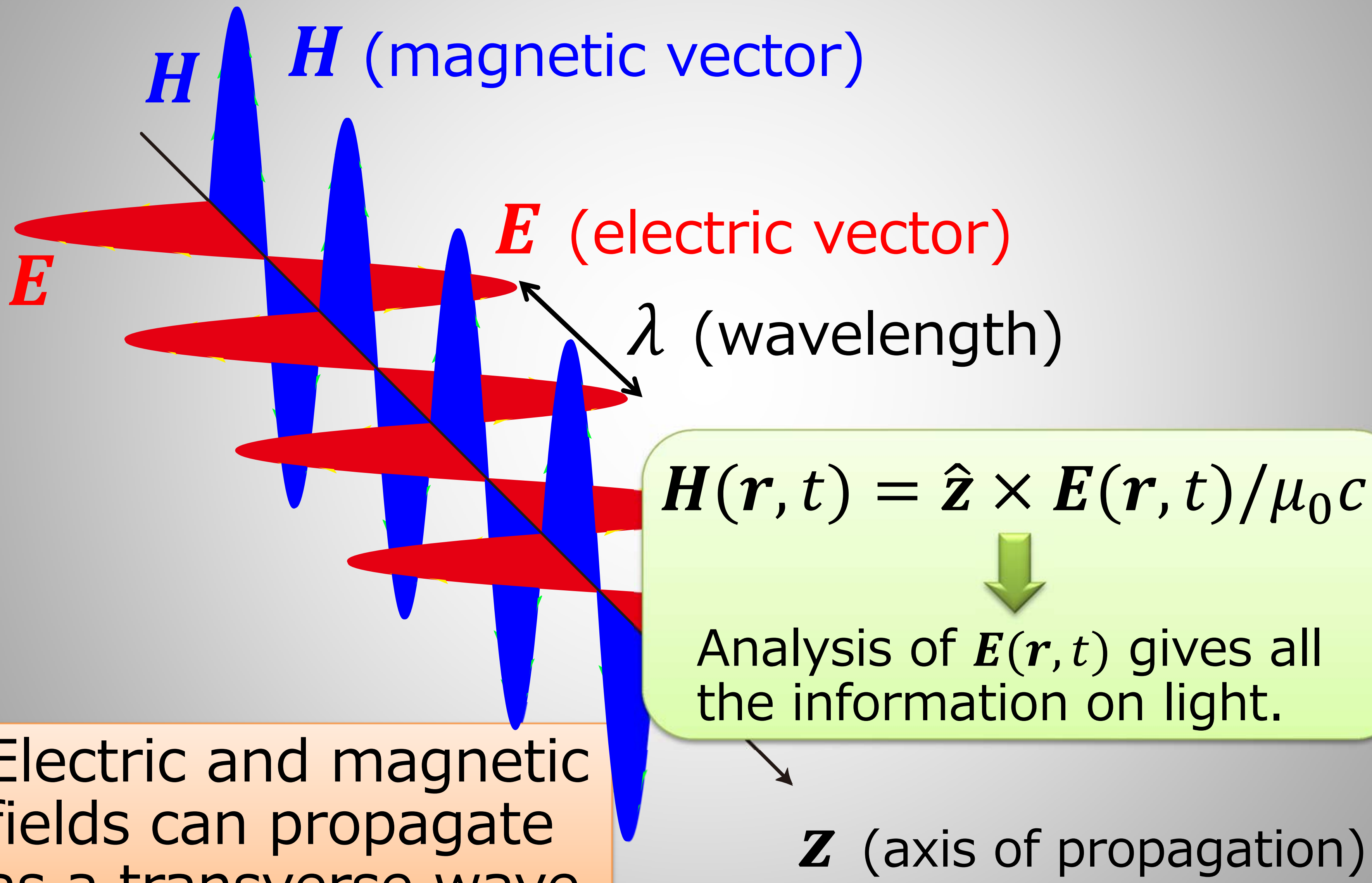
Information on e⁻ (E-field) propagates with the speed of light!

E-Field Line Is Not "Rigid"



Electric field line can propagate as a wave
➡ **Source of Light**

Light as an Electromagnetic Wave



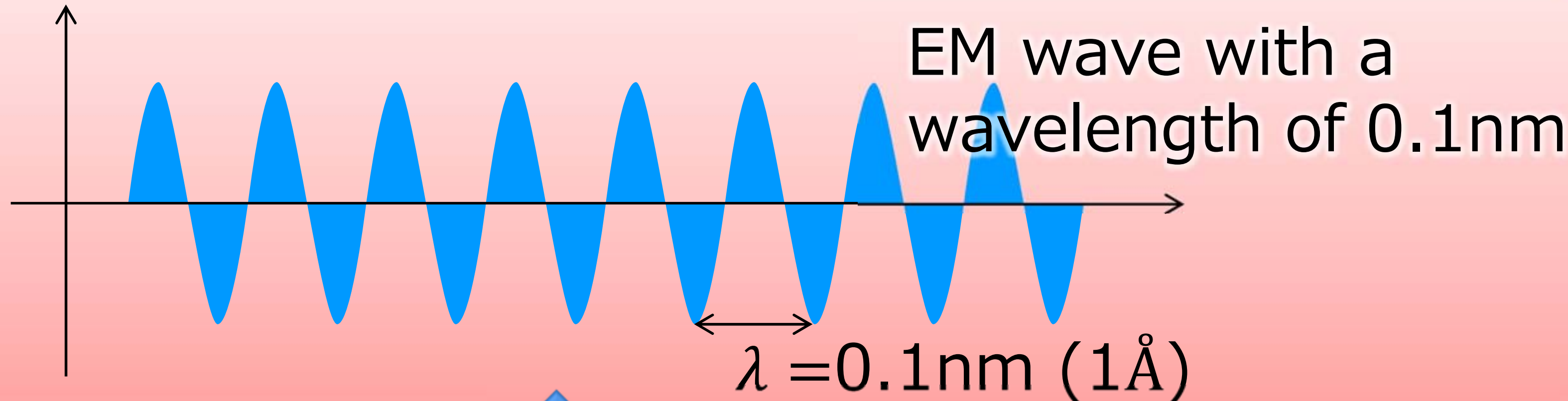
$$\boldsymbol{H}(\boldsymbol{r},t) = \hat{\mathbf{z}} \times \boldsymbol{E}(\boldsymbol{r},t)/\mu_0 c$$

Analysis of $\boldsymbol{E}(\boldsymbol{r},t)$ gives all the information on light.

Electric and magnetic fields can propagate as a transverse wave

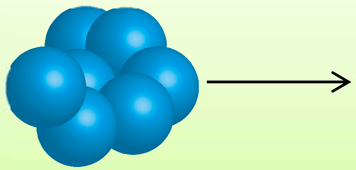
Light as a Photon

Light is not only an electromagnetic wave but also a particle, or a photon.



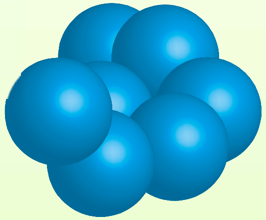
$$E_{photon} = \hbar\omega = hc/\lambda$$

Photons with an energy of 12.4 keV

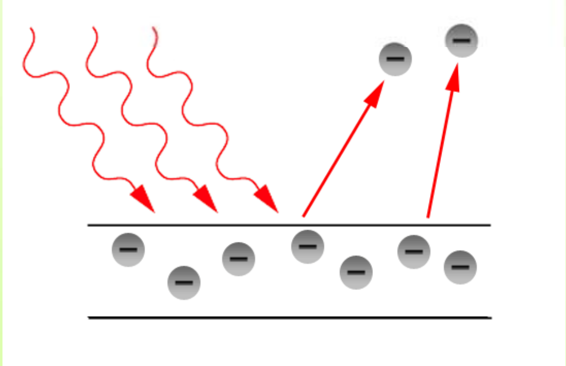


Wave? Photon?

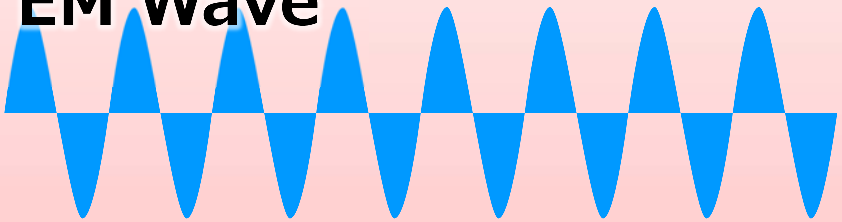
Photon



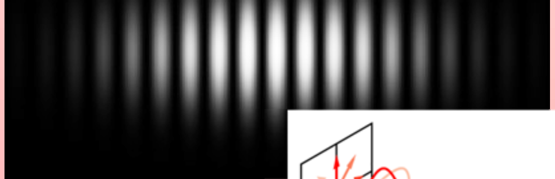
Photoelectric Effect



EM Wave



Interference



Polarization

$$E_{photon} = hc/\lambda$$

- ✓ Evaluation of SR characteristics (brilliance, flux, ⋯)

- ✓ Formulation of SR with classical electrodynamics (field amplitude)

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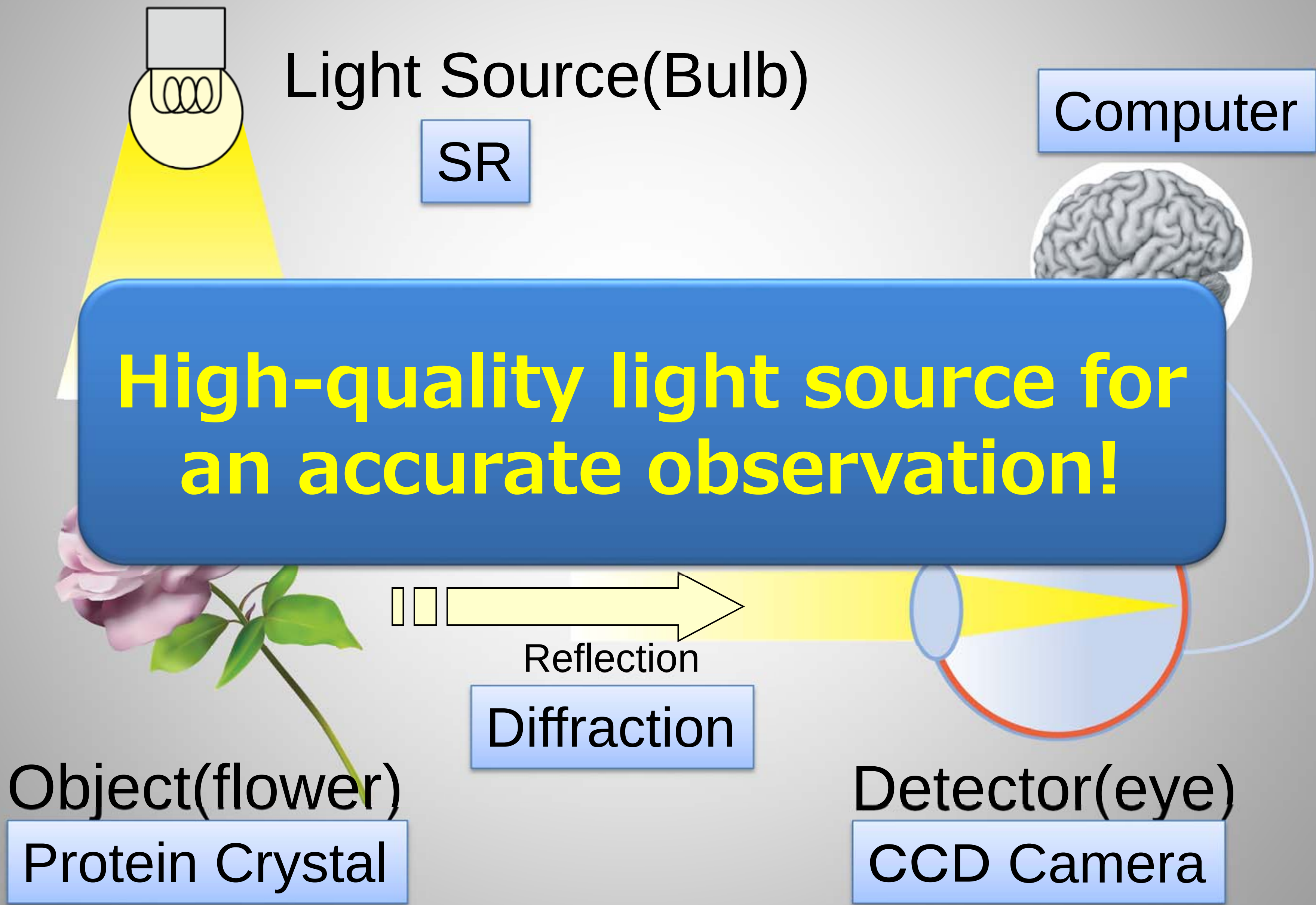
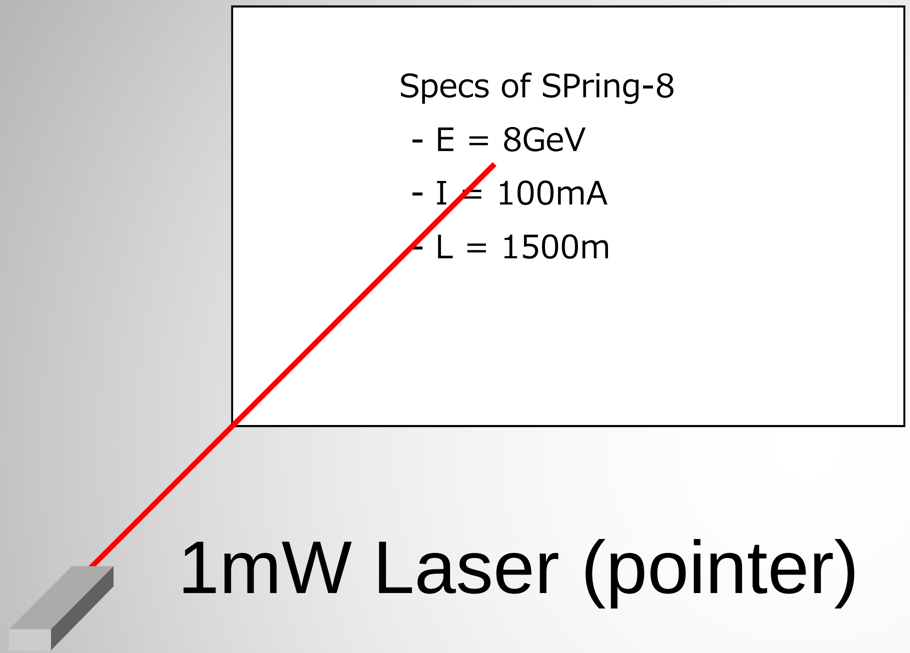
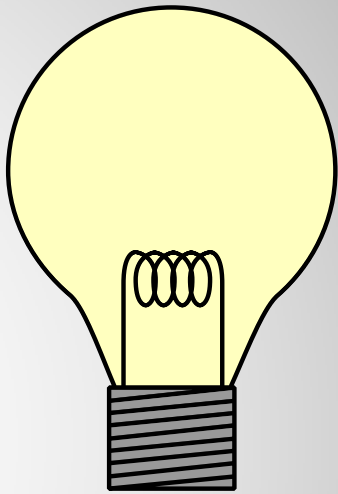
Observation with Light



Which Quality is Better?



1mW Laser (pointer)



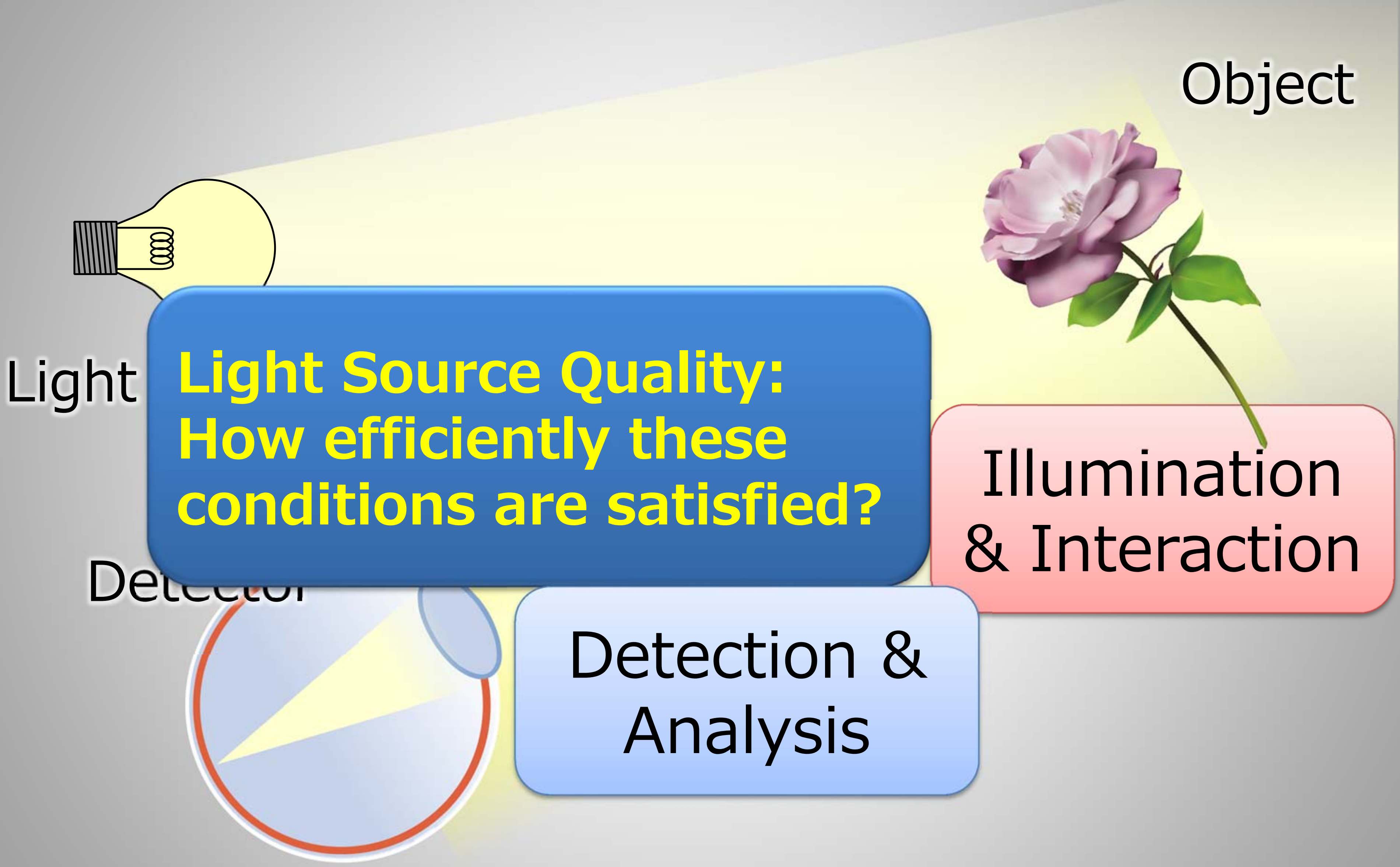
100W Bulb

Lighting equipment in a room: **Bulb**

Pointer during a presentation: **Laser**

Depends on the Object!

How to Define the Quality of Light?(1)



Light Source Quality: How efficiently these conditions are satisfied?

How to Define the Quality of Light?(2)

Important Features of the Light Source

Item	Object		Why?
	Flower	Protein	
Radiation Power	◎	○	# Emitted Photons
Source Size	×	◎	Illuminated Area
Directivity	△	◎	
Monochromaticity	△	◎	Accuracy of Analysis

Brilliance

What is Brilliance ?

$$\text{Brilliance(photons/sec/mm}^2\text{/mrad}^2\text{/0.1\%B.W.)} \sim \frac{\text{Total Power}}{\text{Source Size x Angular Divergence x Band Width}}$$

- Brilliance specifies the quality of light for observation of microscopic objects.
- The brilliance of a light source with a high total power is not necessarily high.

Example of Brilliance Estimation

Item	Bulb	Laser Pointer
Total Power (W)	100	10^{-3}
Angular Div. ($mrad^2$)	$4\pi x10^6$	1
Source Size: (mm^2)	10^2	1
Bandwidth: (%)	100	0.01
Brilliance (photons/sec/....)	$\sim10^8$	$\sim10^{16}$

Laser is the best light source to observe the microscopic object!

X ray as a Probe

- Definition (not unique)
  - Electromagnetic wave (= light) with l of 10 nm(10^{-8} m) ~ 0.1 Å (10^{-11} m)
- Properties
  - High Energy/Photon
  - High Penetration (Roentgen etc..)
- Application to Microscopic Objects
  - X-ray Diffraction
  - Fluorescent X-ray Analysis
- No Practical Lasers!! (until recently)

→ **Synchrotron Radiation (SR)**

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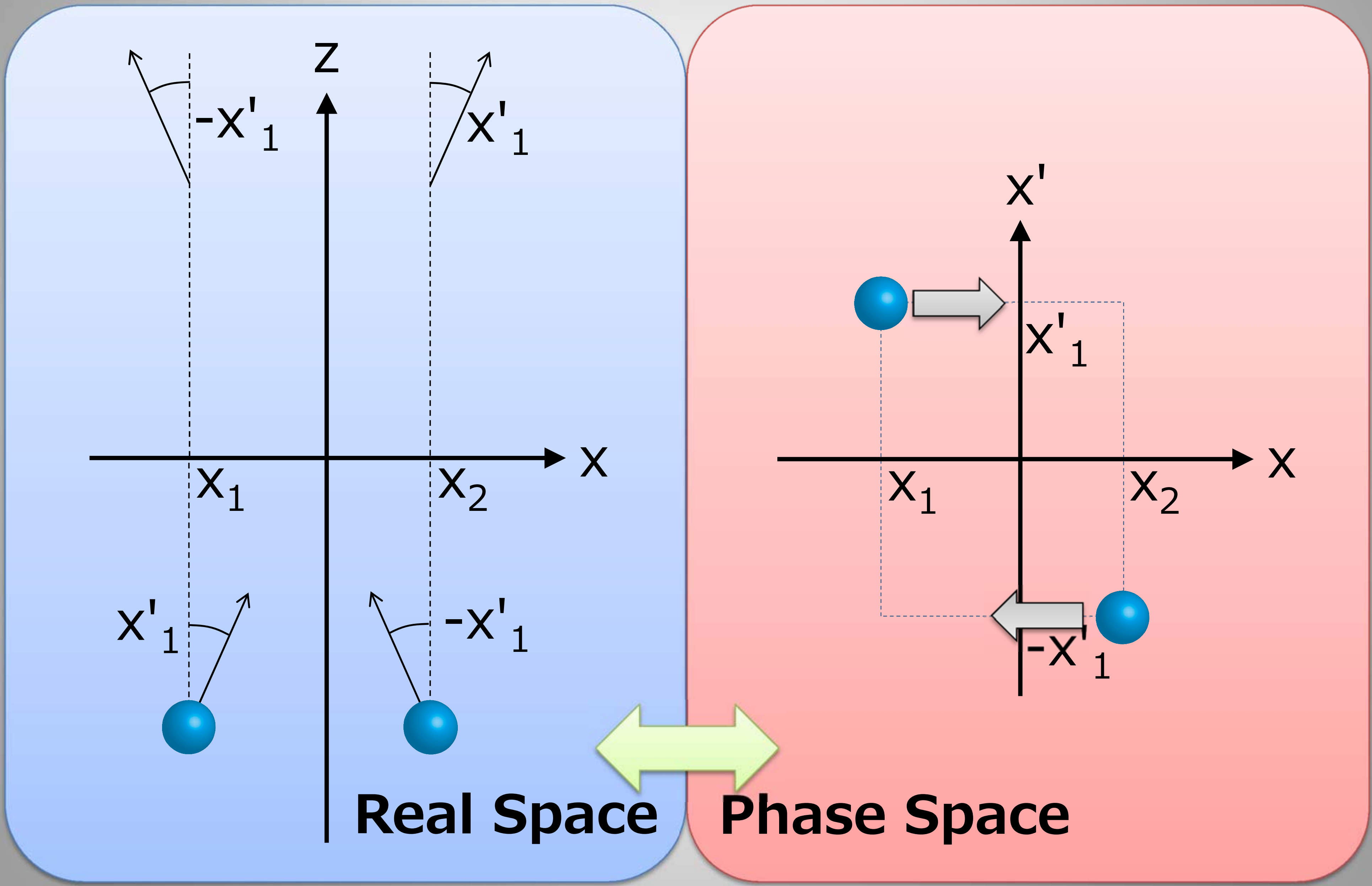
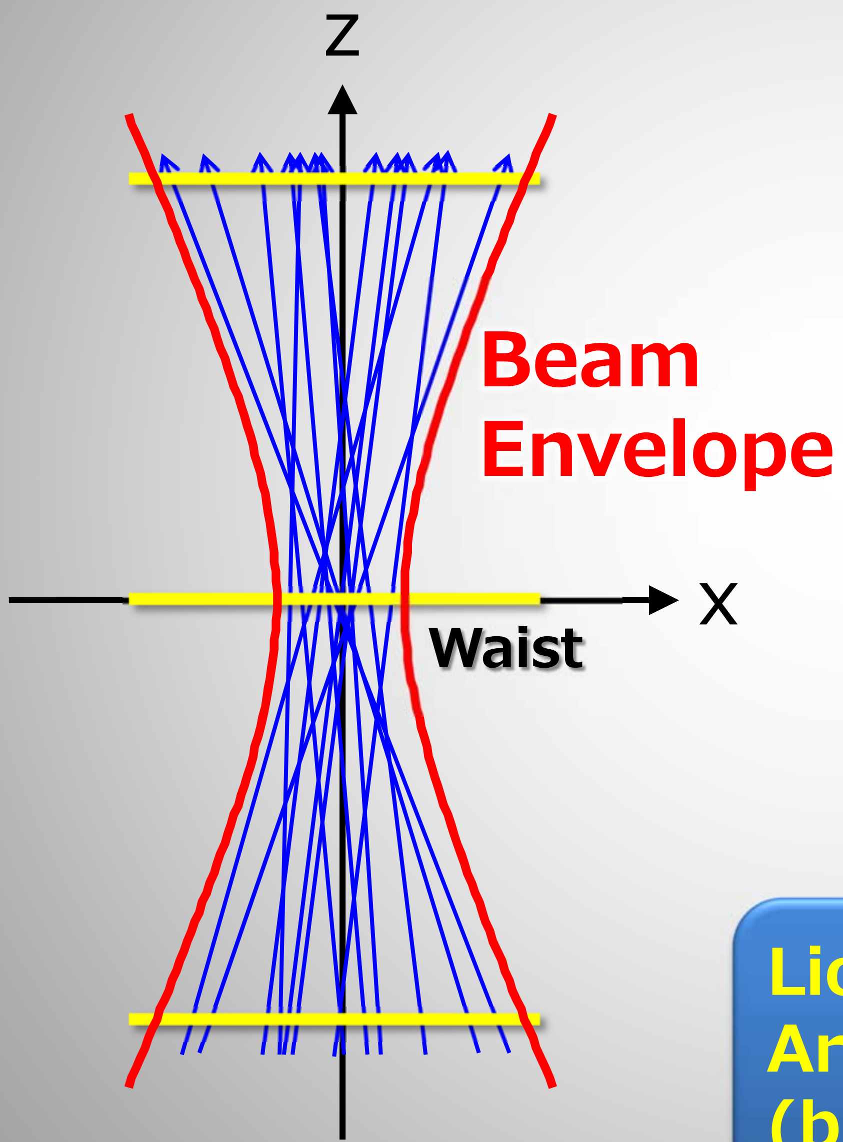
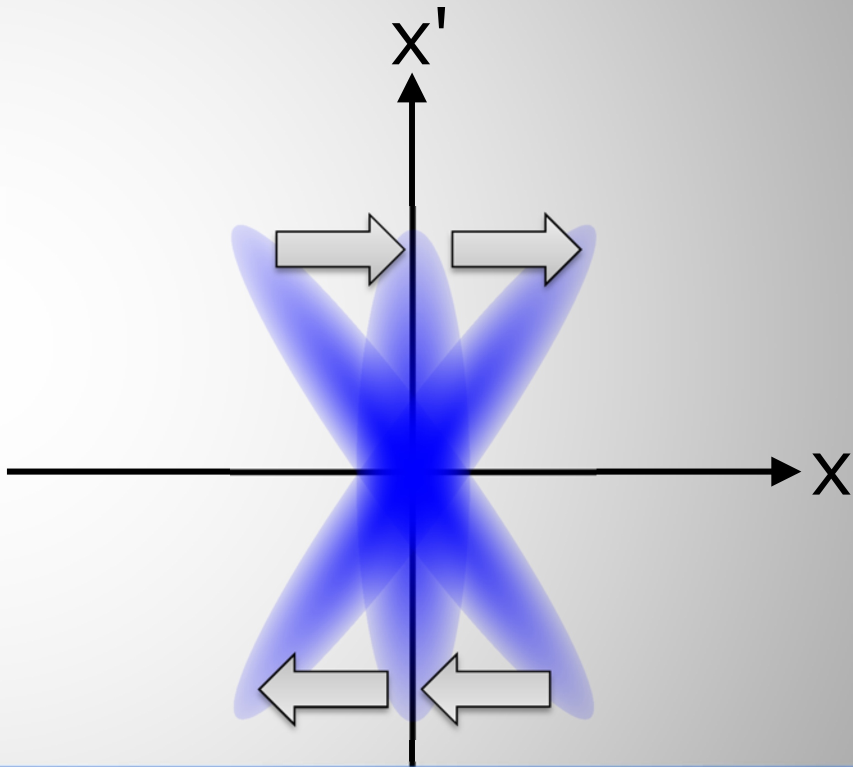
Phase Space Representation



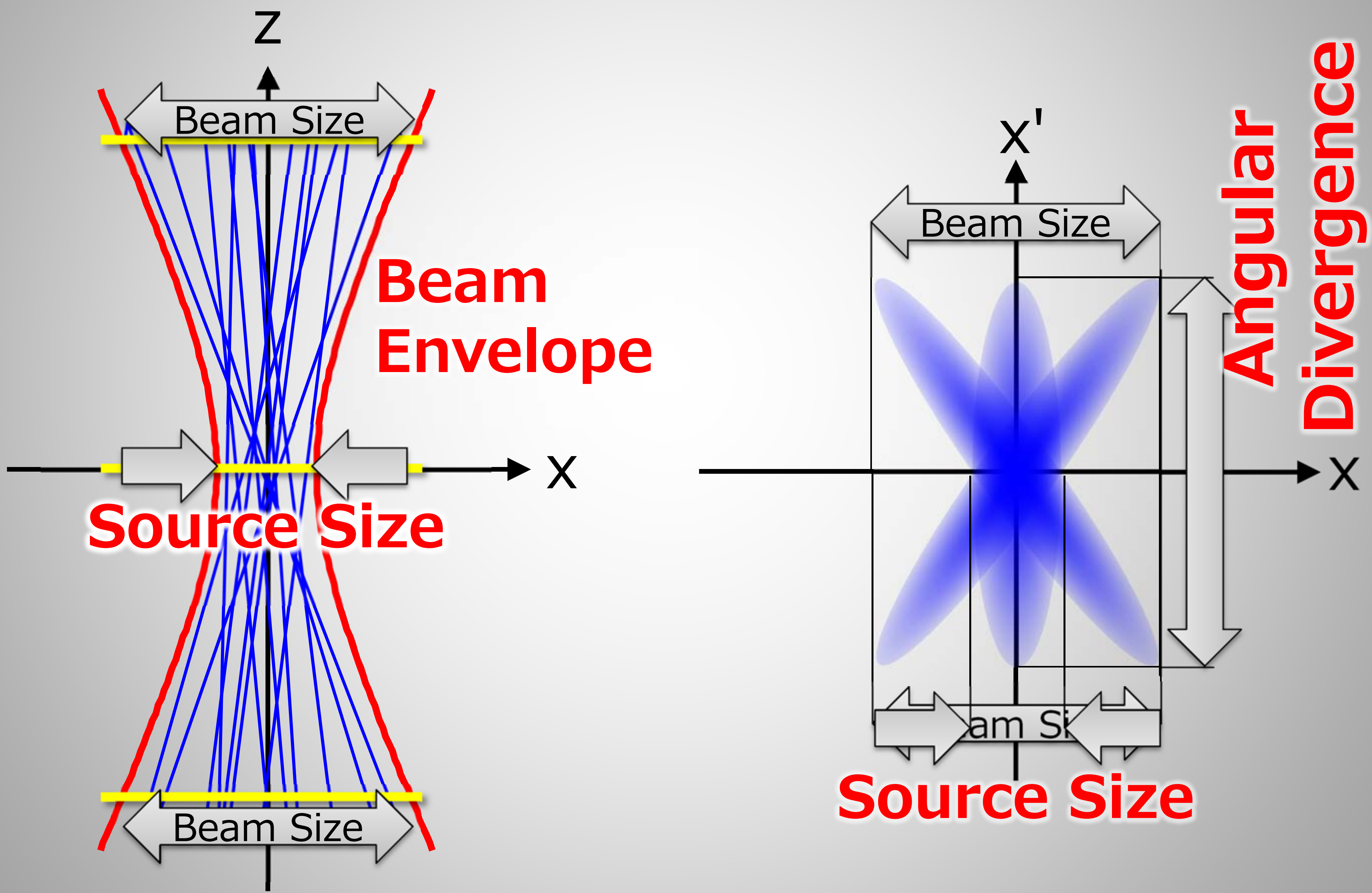
Photon Propagation in Phase Space



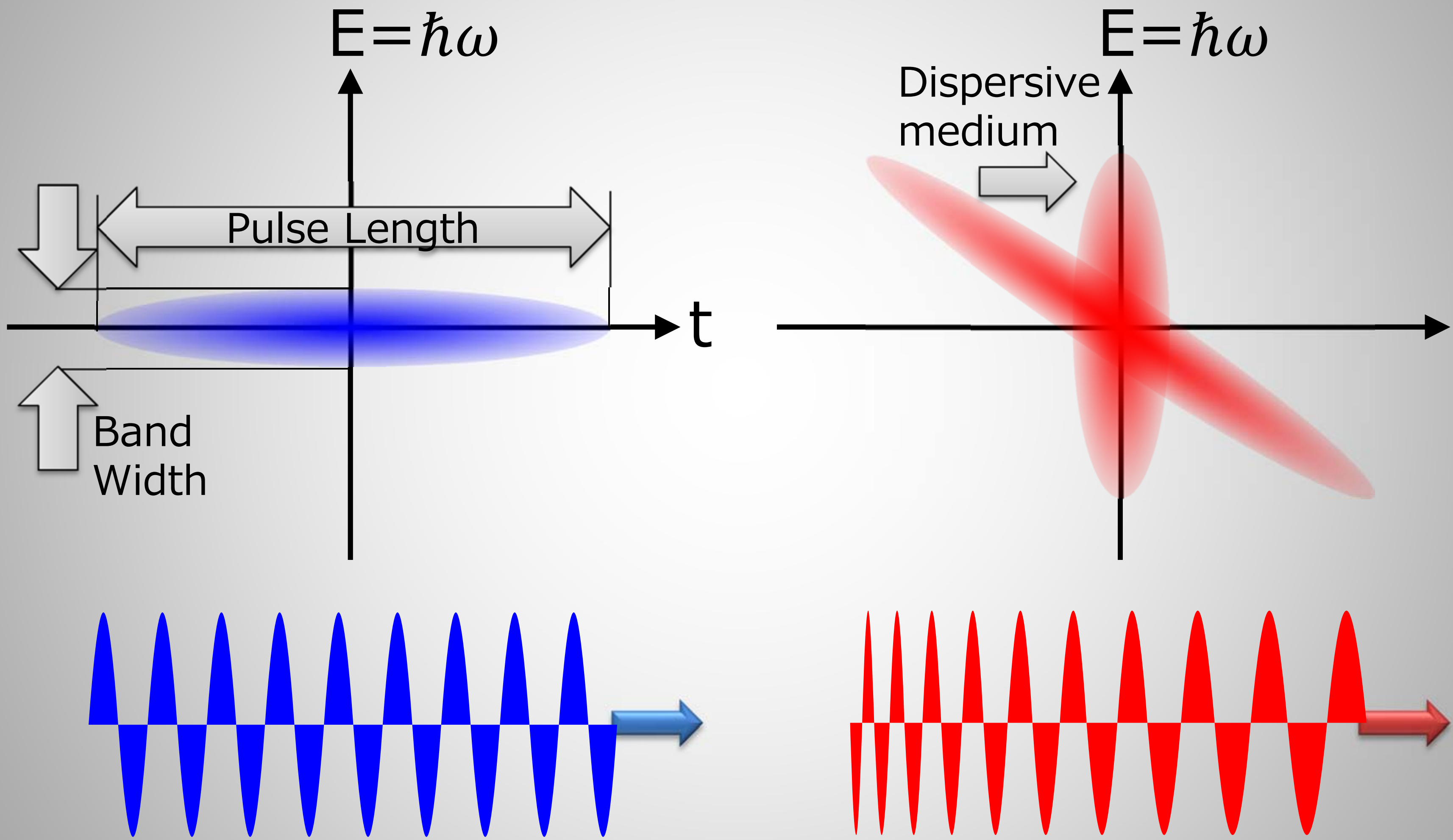


Liouville's Theorem:
Area of the phase ellipse (beam emittance) is constant.

Photon Propagation in Phase Space

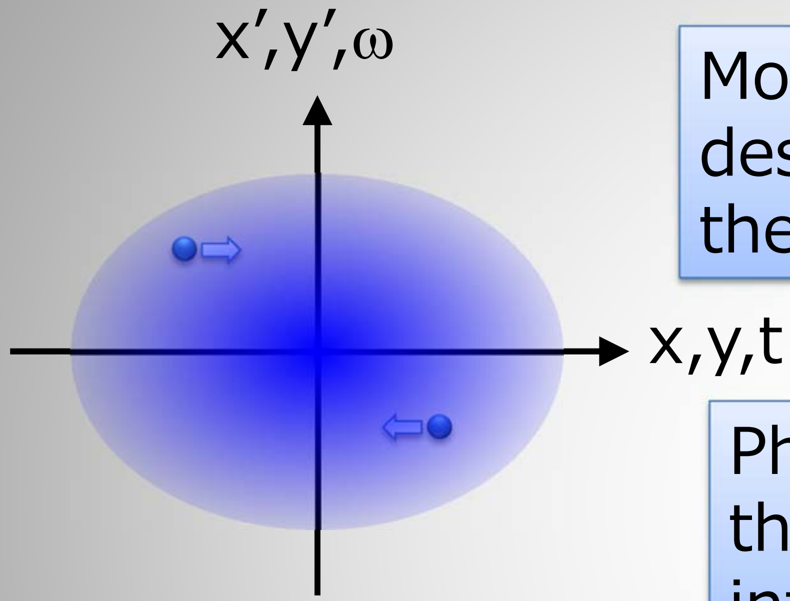


Energy-Time Phase Space



Pulse compression scheme (optical laser)

6D Phase Space & Brilliance



Motion of each photon can be described in a space spanned by the 6 coordinates (phase space)

Photon distribution function in the 6D phase space gives full information on light source

Typical form of the photon distribution

Central Freq.

$$n(x,x',y,y',t,\omega) = \boxed{B_0}\exp\left[-\frac{x^2}{2\sigma_x{}^2}-\frac{y^2}{2\sigma_y{}^2}-\frac{x'^2}{2\sigma_{x'}{}^2}-\frac{y'^2}{2\sigma_{y'}{}^2}-\frac{t^2}{2\sigma_t{}^2}-\frac{(\omega-\omega_c)^2}{2\sigma_\omega{}^2}\right]$$

Brilliance

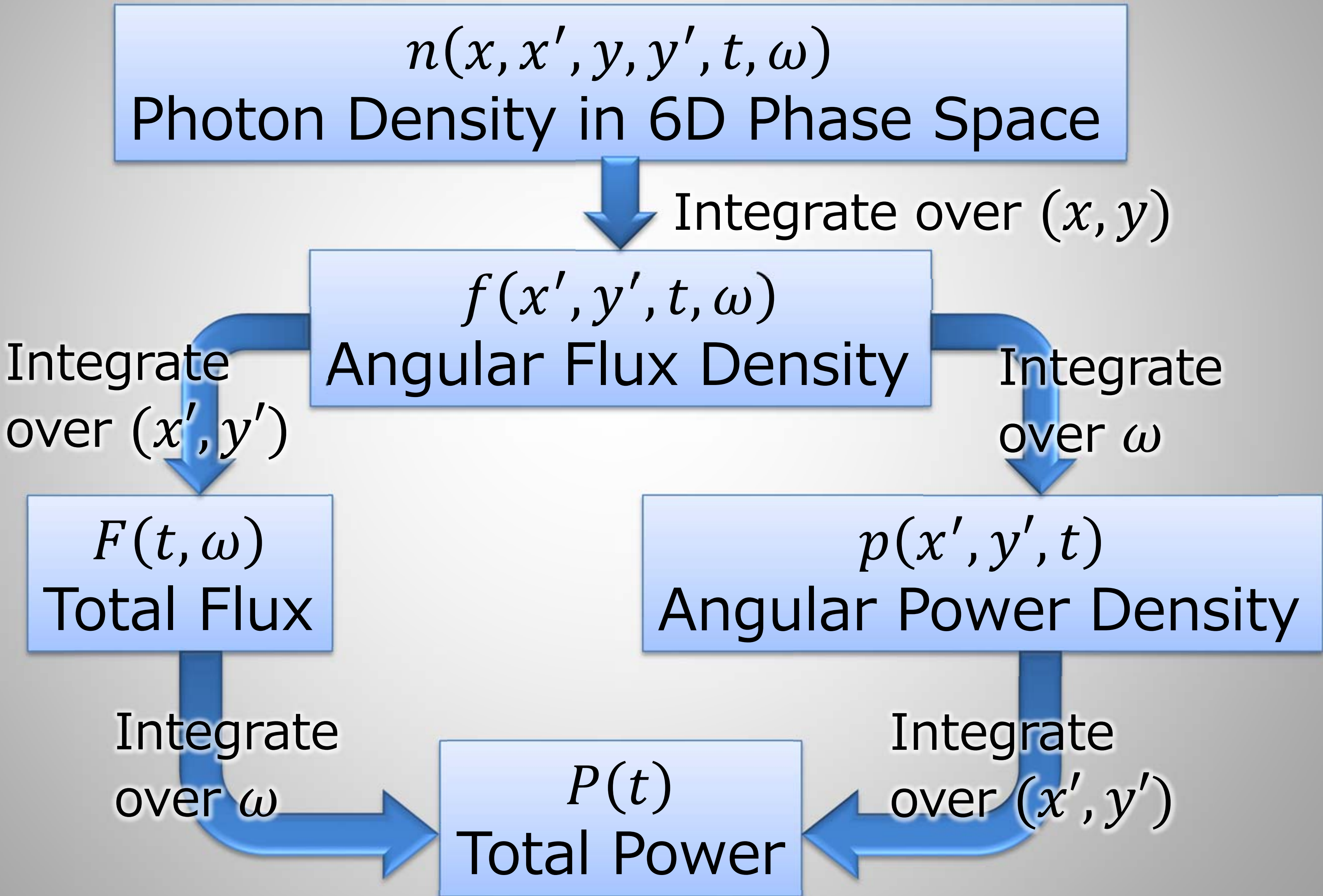
Source Size

Angular Divergence

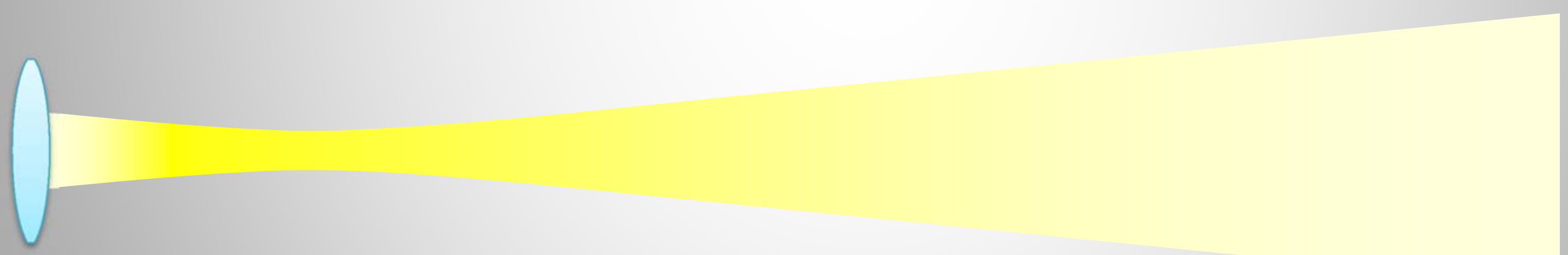
Pulse Length

Band Width

Photon Flux & Radiation Power



To Be More Specific…



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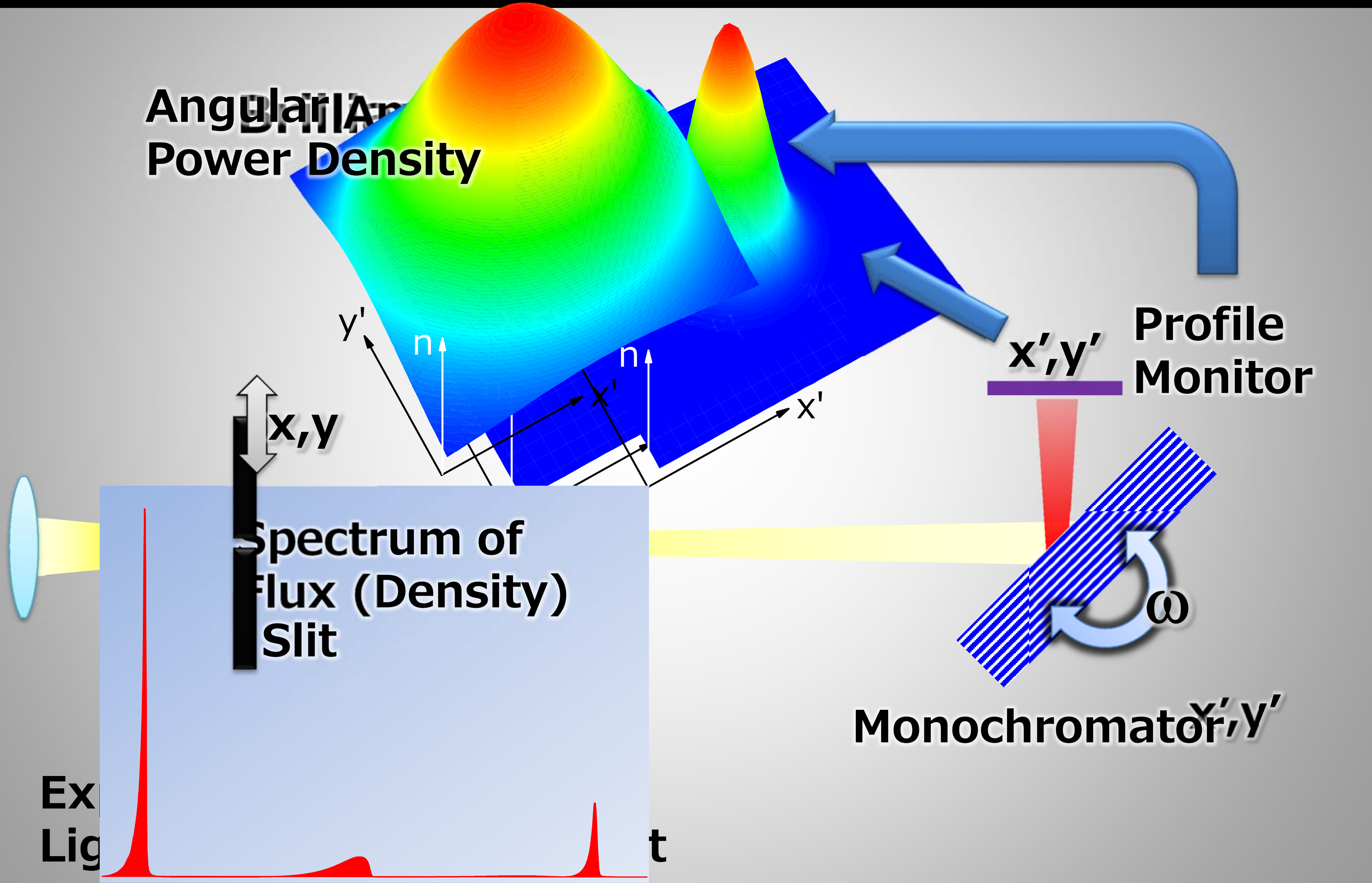

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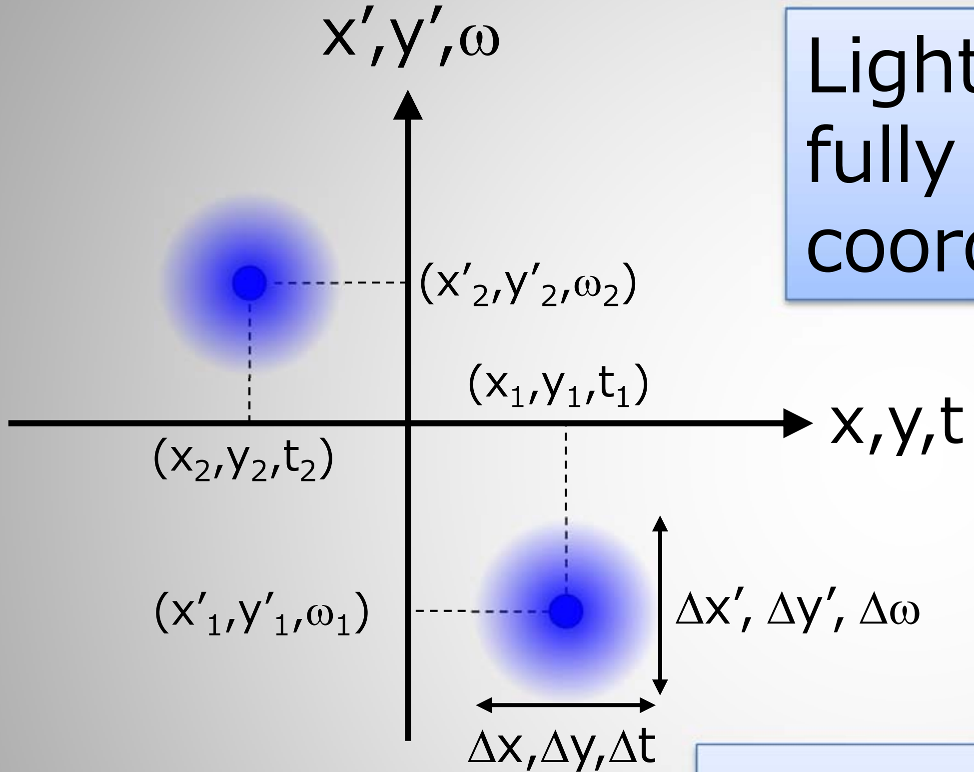
Uncertainty of Light

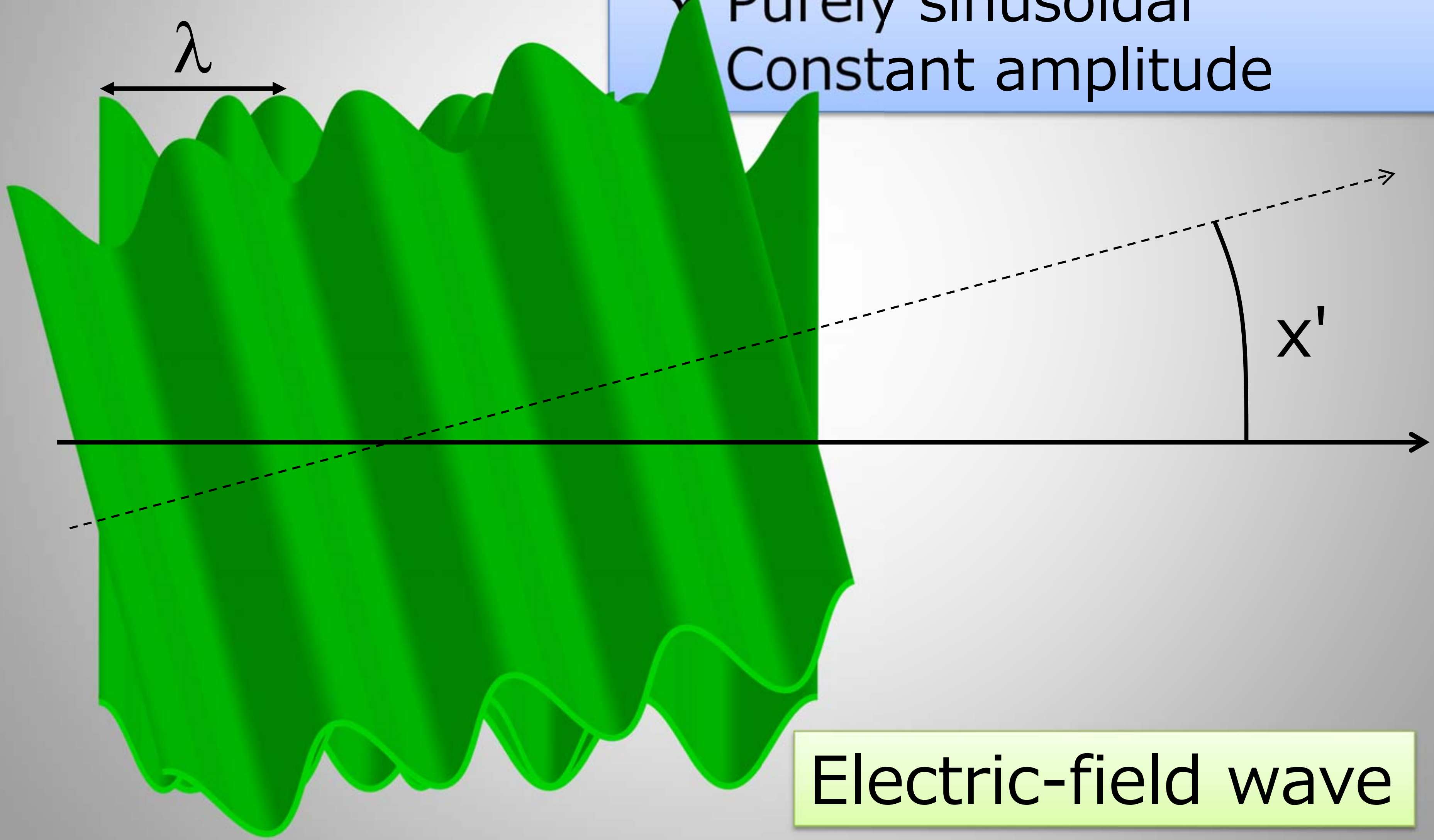


Light as a photon can be fully described by the 6 coordinate variables.

Because of the **wave nature**, they have some uncertainty characterized by the so-called Fourier transform.

Monochromatic & Plane Wave

Monochromatic & Plane Wave
- ✓ Purely sinusoidal
- ✓ Constant amplitude

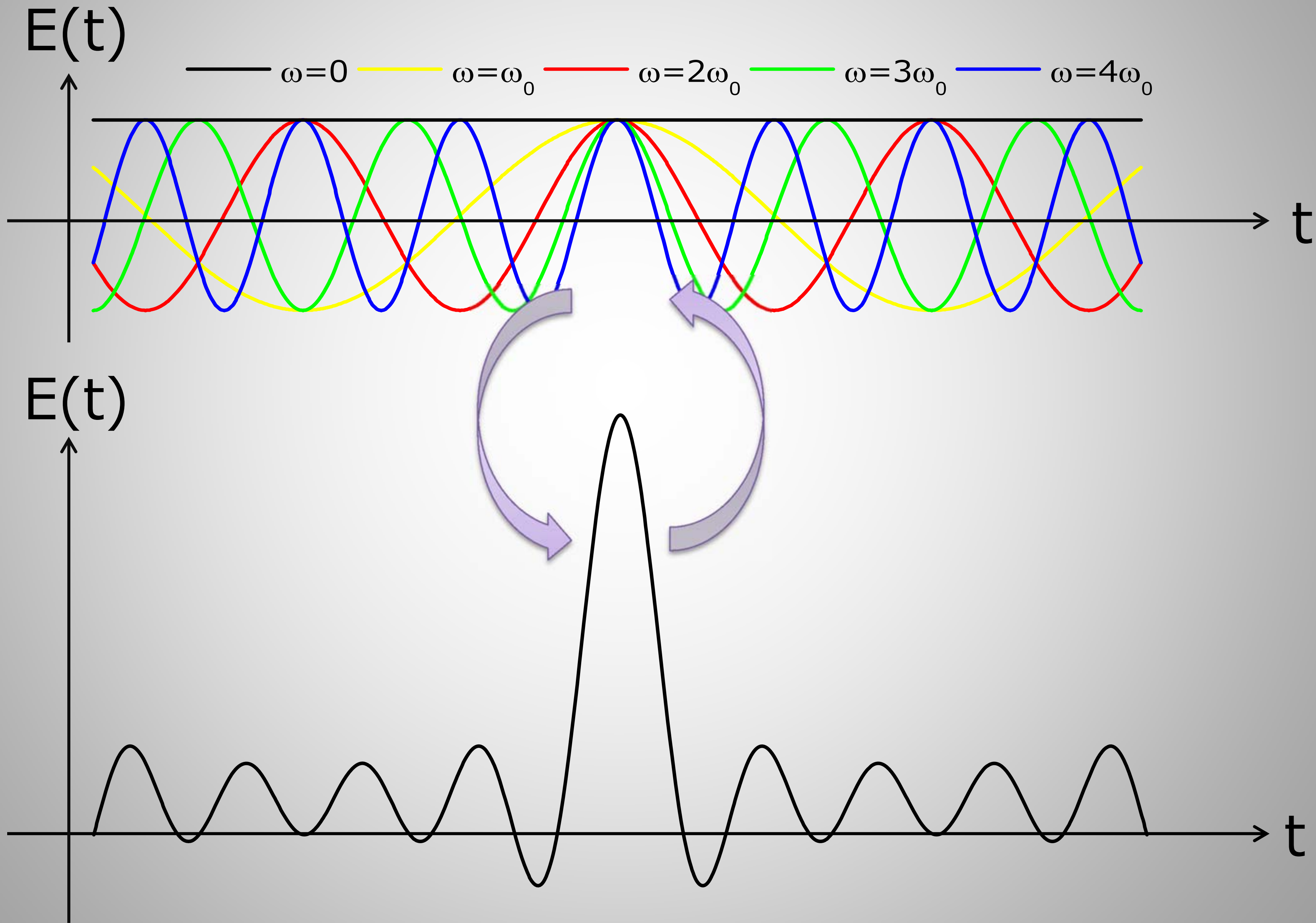


Electric-field wave

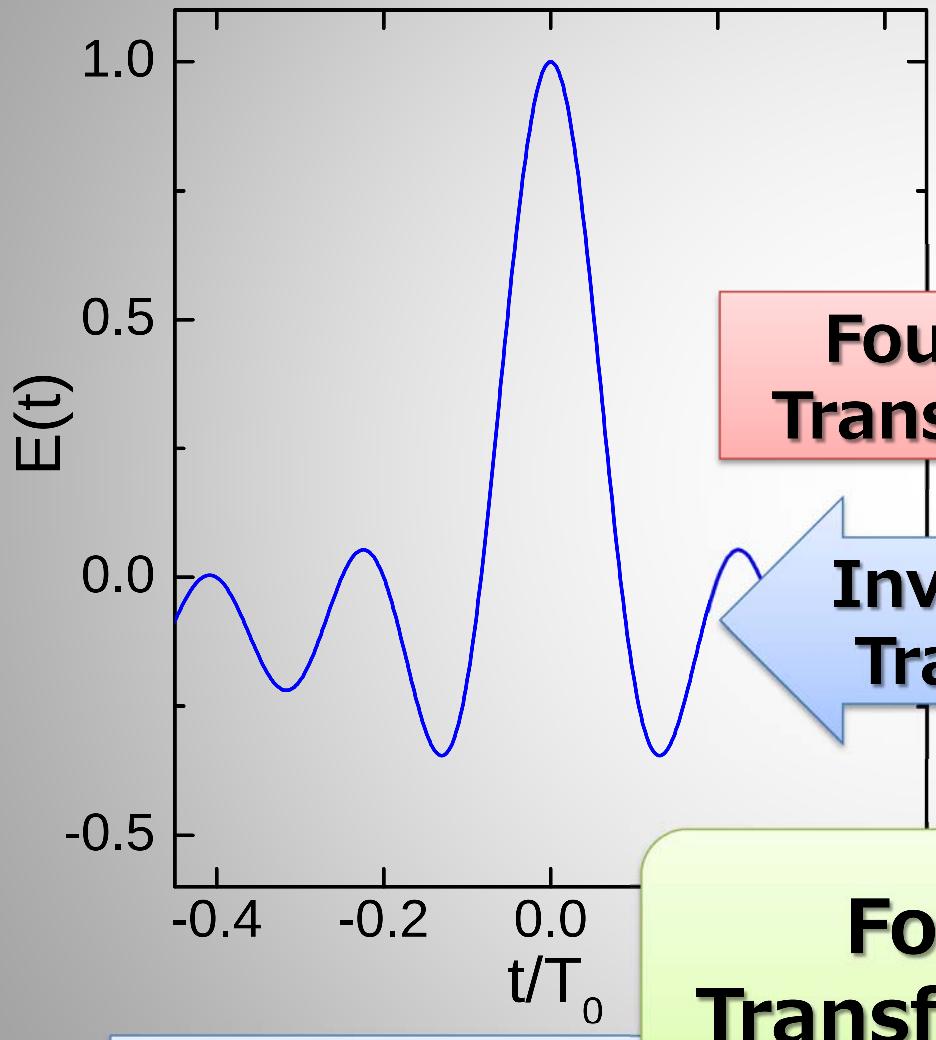
In Reality…

- A monochromatic and plane wave is an (ideal) form of a wave.
- In practice, a wave is composed of many ideal waves having different λ and x'.
- **Fourier Transform** is a mathematical operation to decompose an arbitrary wave into a number of monochromatic and plane waves.

Composition of Monochromatic Waves



Time & Frequency Domains



Fourier Transform

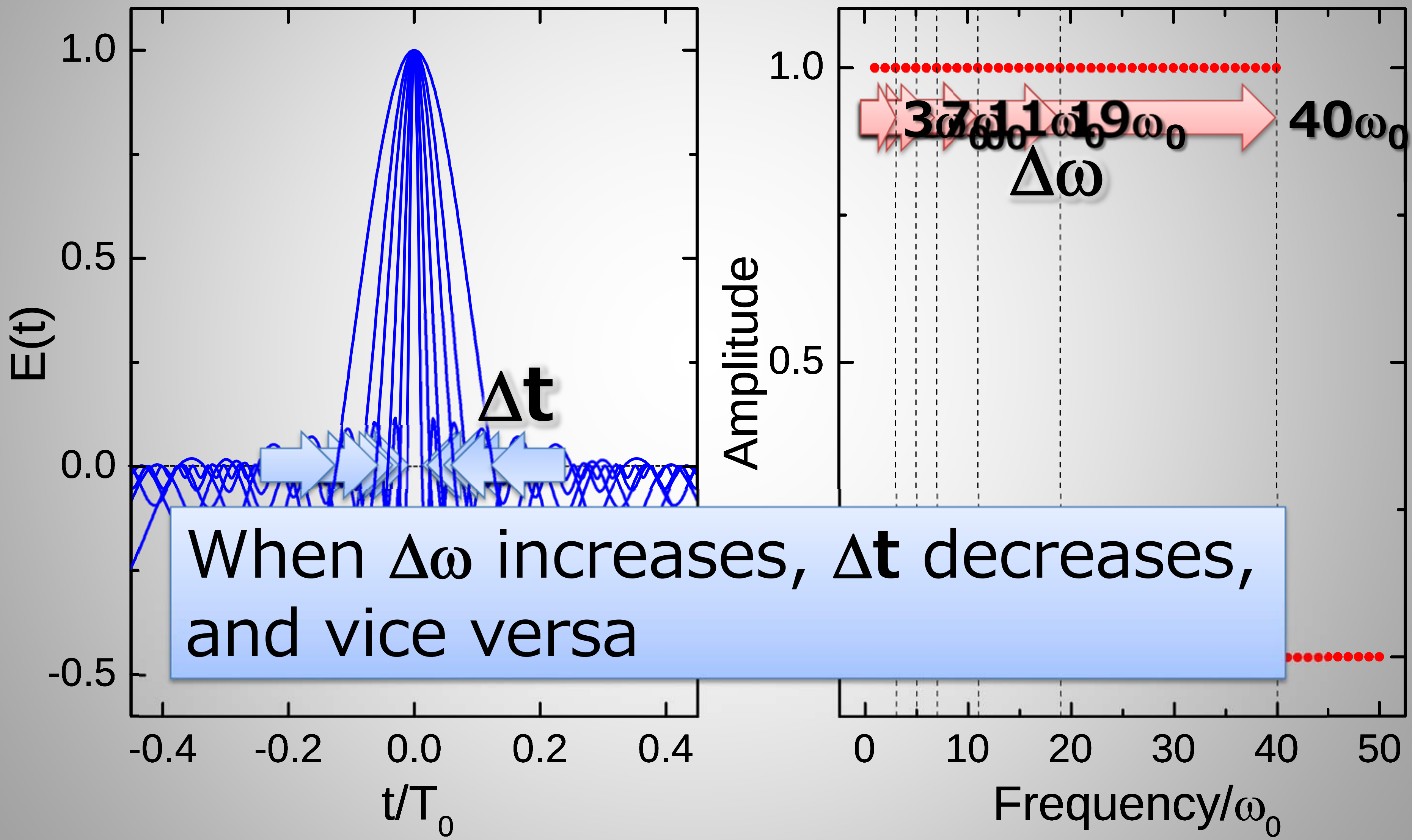
Inv. Fourier Transform

Fourier Transform Pair

Time Domain: wave
a function of t

cy Domain: wave
amplitude as a function of ω

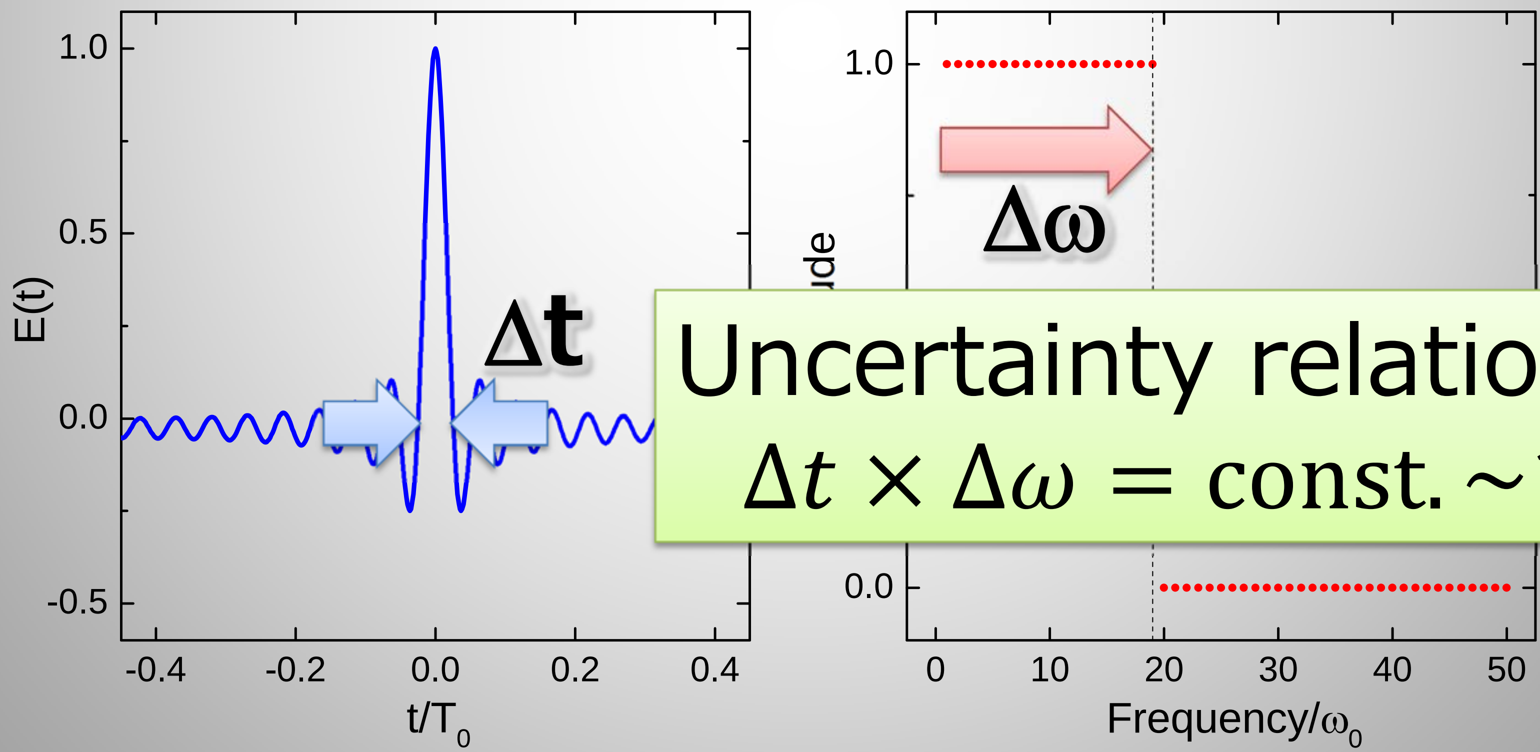
How Does the Waveform Changes?



When $\Delta\omega$ increases, Δ**t** decreases, and vice versa

Uncertainty Relation

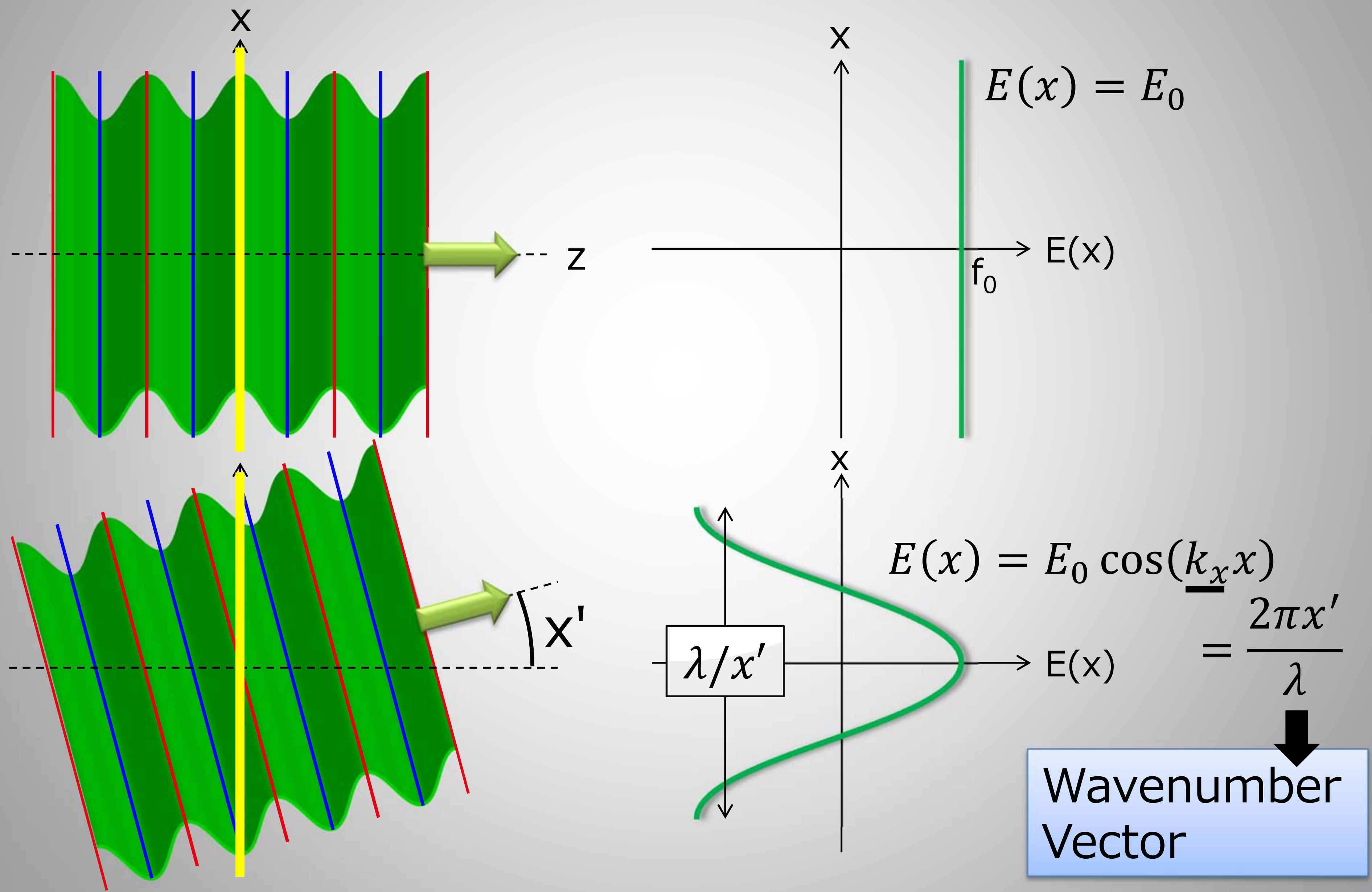
- Δt and Δω are regarded as uncertainty of a photon
  - Δt: longitudinal position (pulse width)
  - Δω: photon energy (bandwidth)



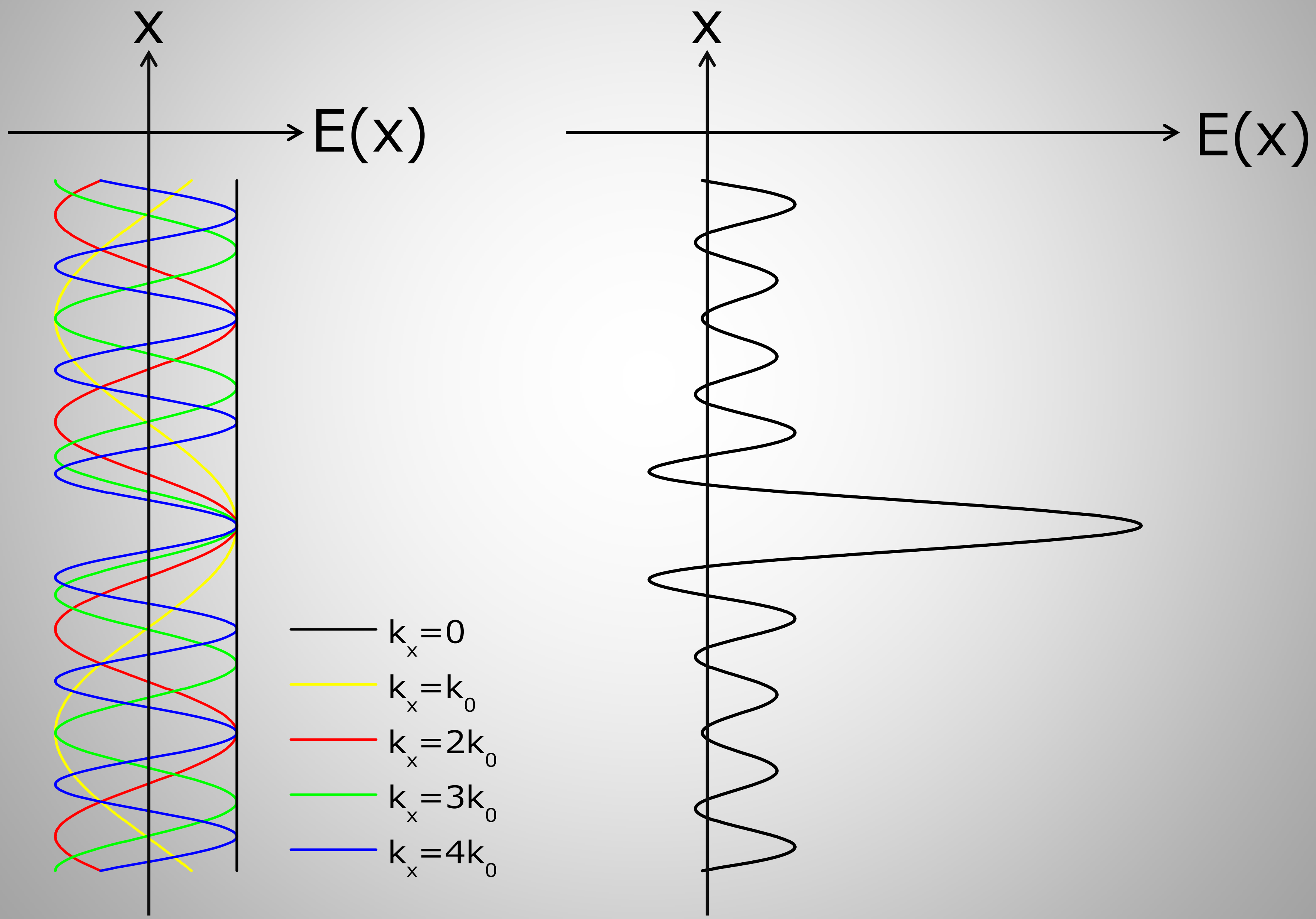
Uncertainty relation:

$$\Delta t \times \Delta\omega = \mathrm{const.} \sim 1$$

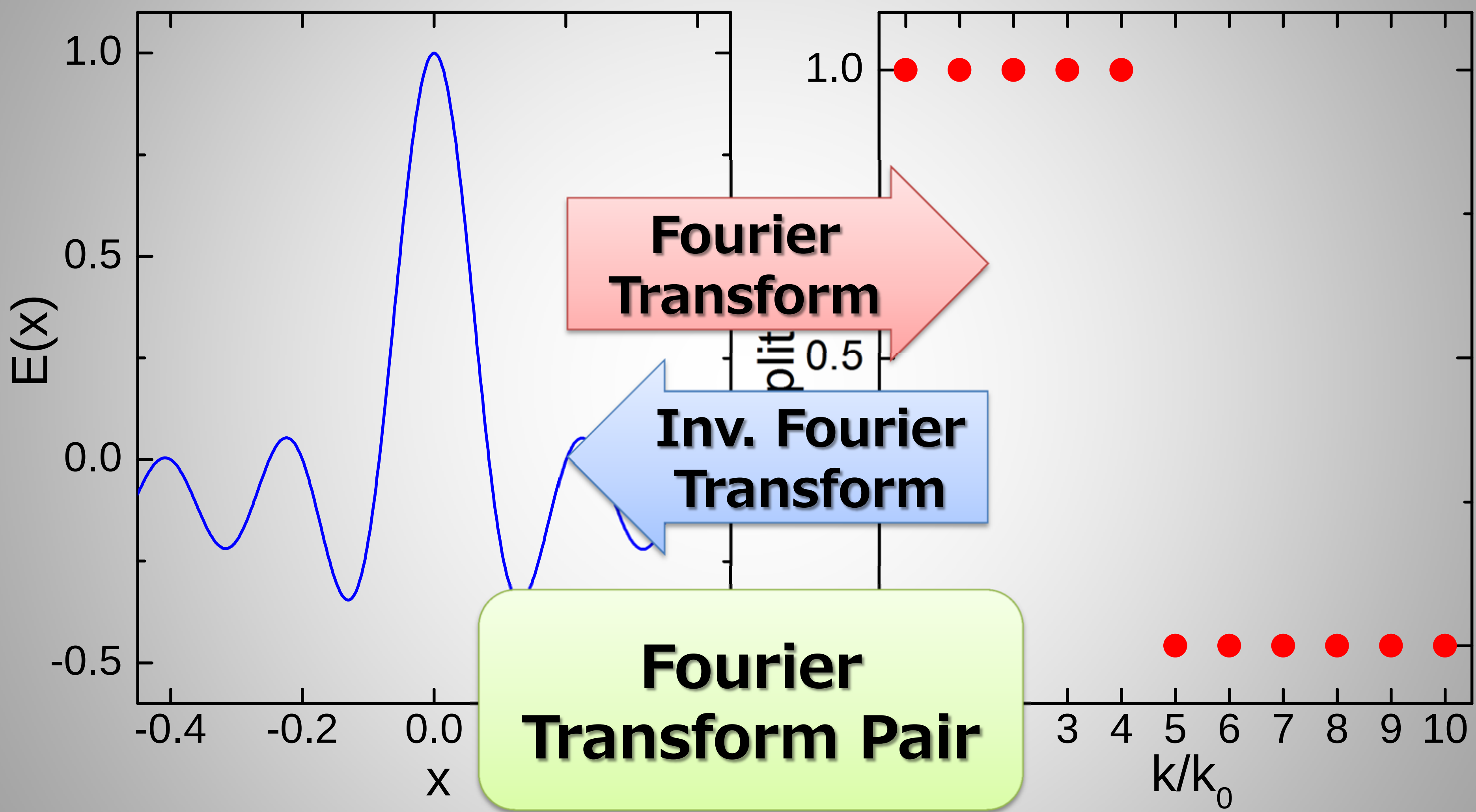
Composition of Plane Waves



Composition of Plane Waves



Spatial & Angular Domains



Spatial Domain: waveform as a function of x

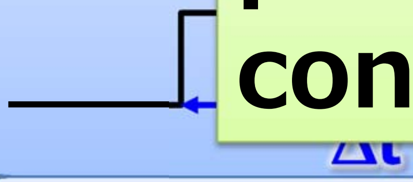
Angular Domain: wave amplitude as a function of k

Fourier Transform Examples

$F(t)$ **Fourier Transform** → $\tilde{F}(\omega) = \int F(t) e^{i\omega t} dt$

Boxcar

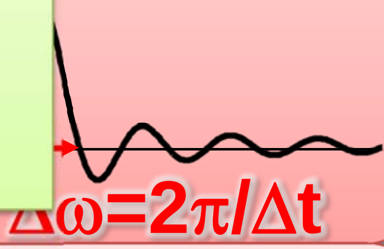
$$\begin{cases} 1/\Delta t \\ 0 \end{cases}$$



Product of "widths" of Fourier transform pair (F and $\tilde{F}$) is a constant

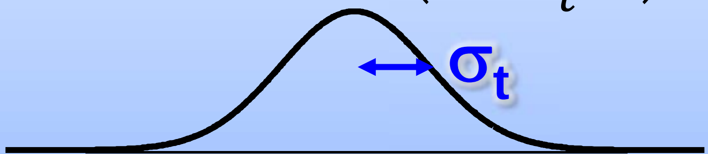
" Function

$\mathrm{sinc}(\omega\Delta t/2)$



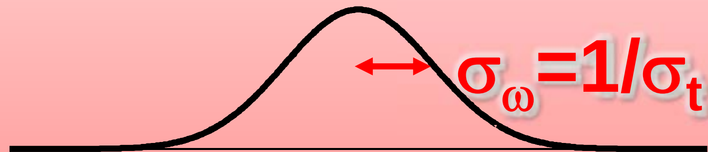
Gaussian Function

$$\frac{1}{\sqrt{2\pi}\sigma_t} \exp\left(-\frac{t^2}{2\sigma_t{}^2}\right)$$



Gaussian Function

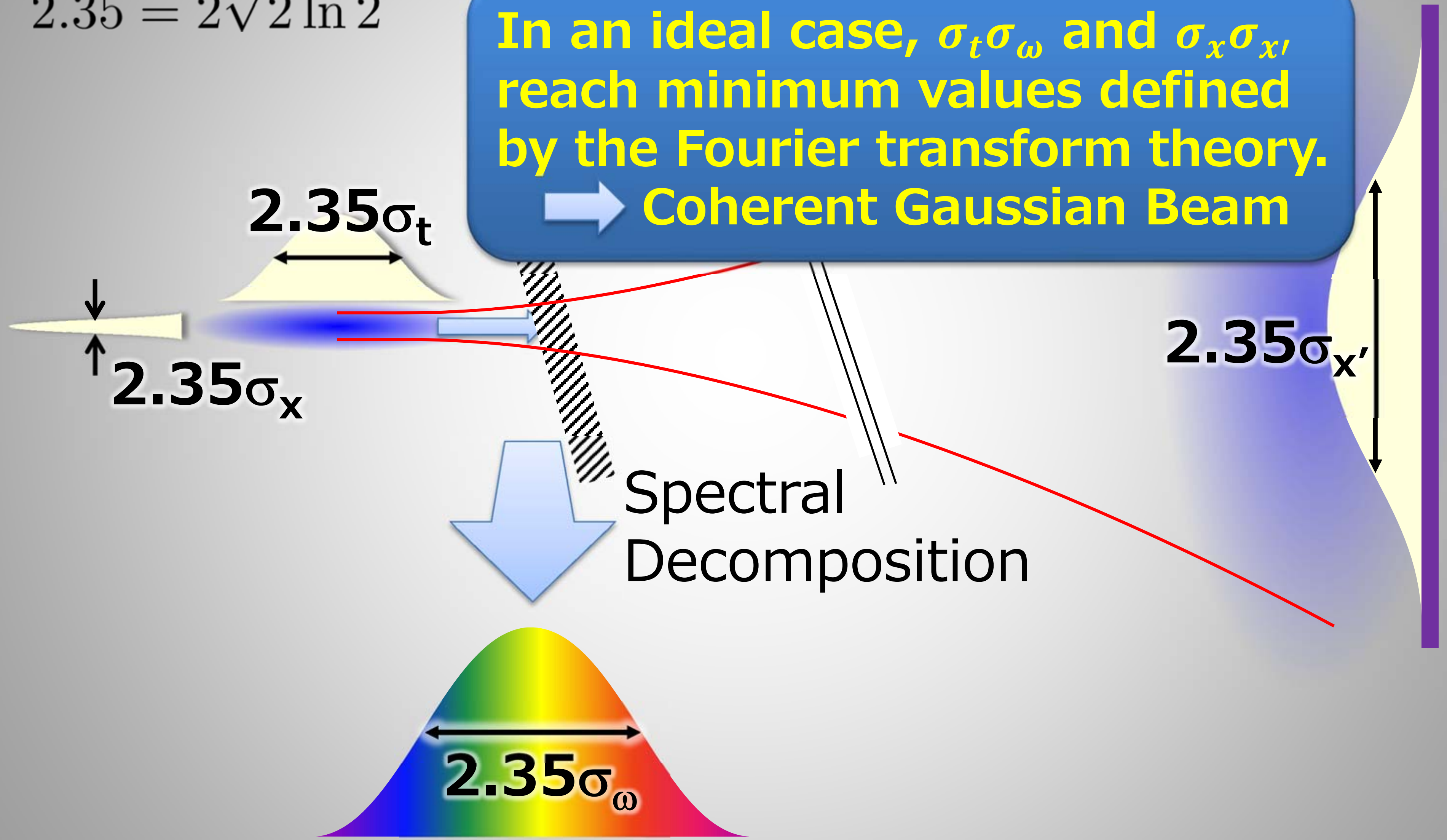
$$\exp\left(-\frac{\omega^2 \sigma_t{}^2}{2}\right)$$



Coherent Gaussian Beam

$$2.35 = 2\sqrt{2\ln 2}$$

In an ideal case, $\sigma_t\sigma_\omega$ and $\sigma_x\sigma_{x'}$ reach minimum values defined by the Fourier transform theory.
⇨ Coherent Gaussian Beam



Fourier and Diffraction Limits

	Condition	Extreme Cases	
Fourier Limit	a) temporal $\sigma_t \sigma_\omega \geq \frac{1}{2}$	$\sigma_t = 0$ $\sigma_\omega = \infty$	White (or Pulse) Light
		$\sigma_t = \infty$ $\sigma_\omega = 0$	Monochromatic Light
Diffraction Limit	b) spatial $\sigma_x \sigma_{x'} \geq \frac{\lambda}{4\pi}$ $\sigma_y \sigma_{y'} \geq \frac{\lambda}{4\pi}$	$\sigma_x = \infty$ $\sigma_{x'} = 0$	Parallel Light
		$\sigma_x \sim \lambda$ $\sigma_{x'} \sim 1$	Minimum Focal Size

- If equality a) holds, the light is:
  - **Temporally Coherent** or **Fourier Limited**
- If equality b) holds, the light is:
  - **Spatially Coherent** or **Diffraction Limited**

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SR: Light from a Moving Electron

- Unlike the ordinary light source (sun, light bulb,...), the light emitter of SR (electron) is ultra-relativistic.
- The characteristics of SR is thus quite different because of relativistic effects.
- What we have to take care is:
  1. Speed-of-light limit
  2. Squeezing of light pulse
  3. Conversion of the emission angles

Speed-of-Light Limit

Within the framework of relativity, the velocity of any object never exceeds the speed of light.

$$v/c = \beta = \sqrt{1-\gamma^{-2}}$$

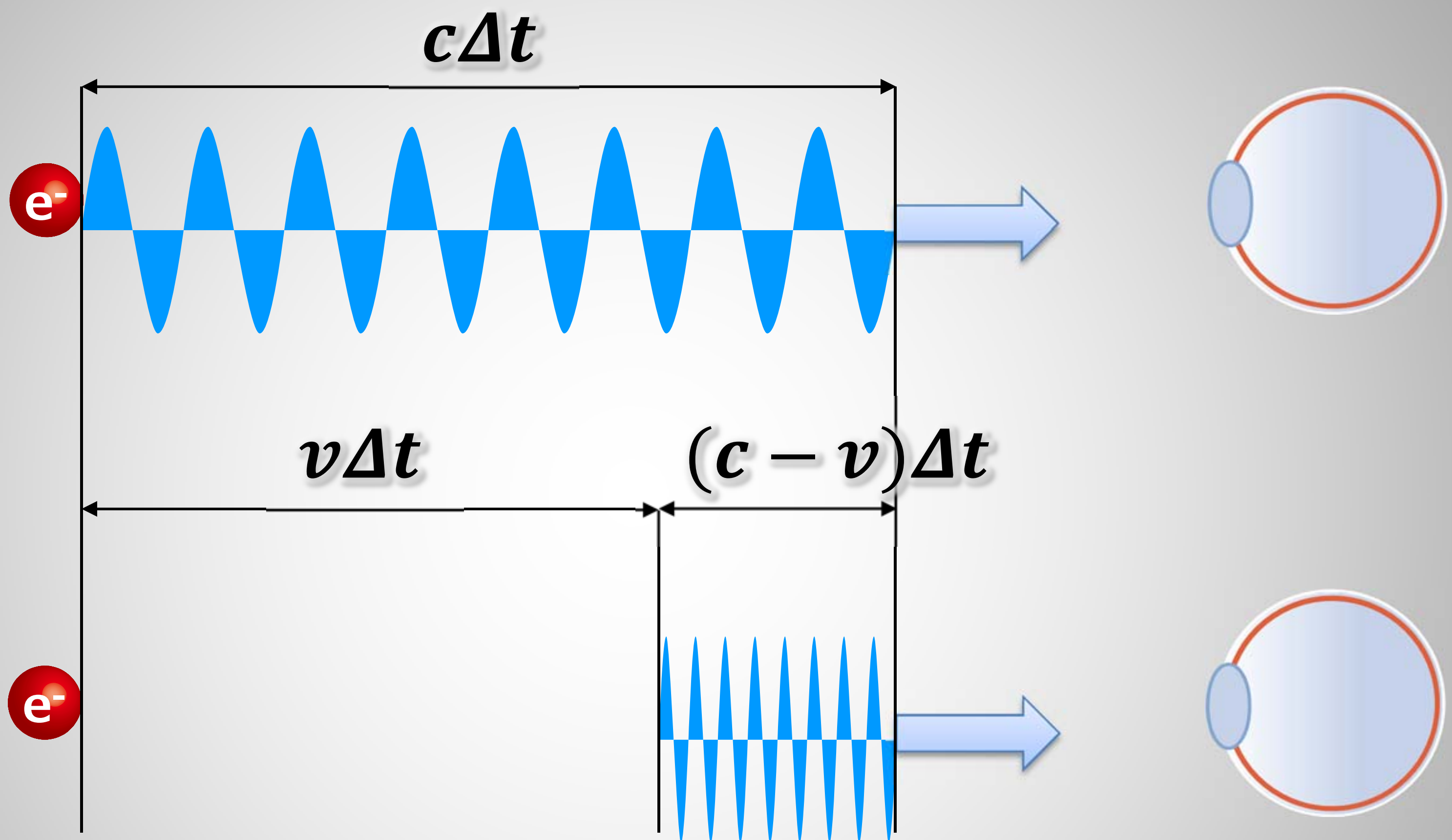
$$\sim 1 - \frac{1}{2\gamma^2}$$

$$\gamma = \frac{E}{mc^2}$$

:Lorentz Factor
(relative electron energy, mc^2=0.511MeV)

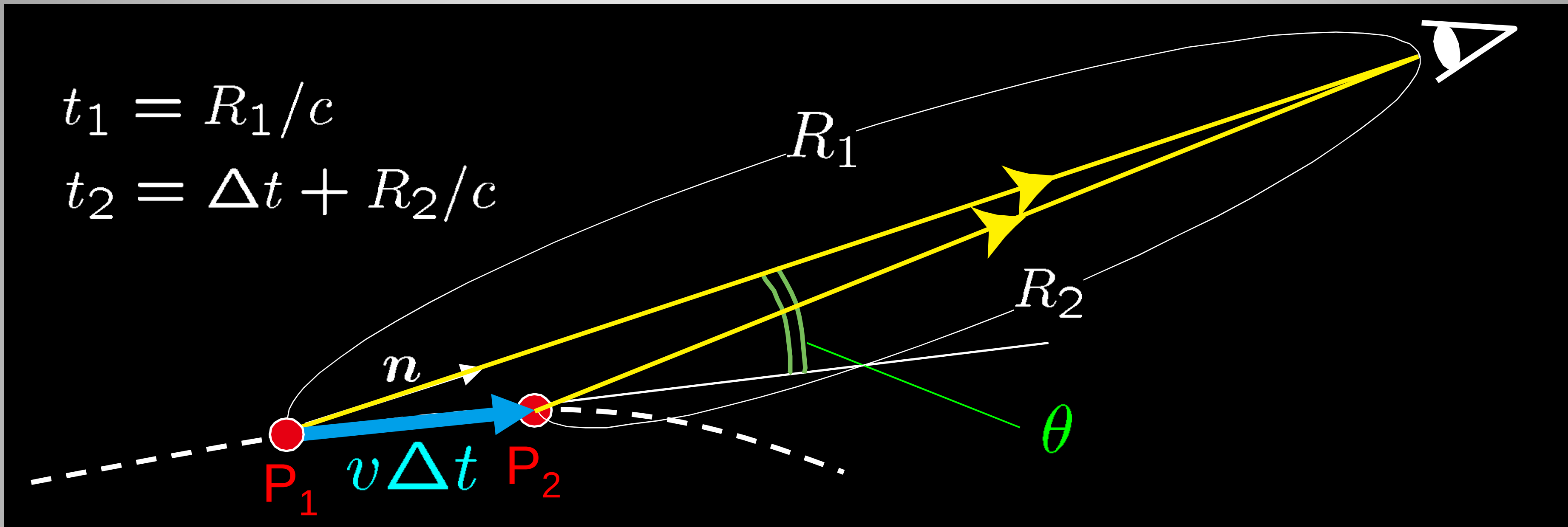
Energy	β
1MeV	0.941
10MeV	0.9988
100MeV	0.999987
8GeV	0.999999998

Squeezing of Light Pulse Duration



$$\text{Pulse Duration Ratio} = \frac{(c-v)\Delta t}{c\Delta t} = 1 - \beta \sim \frac{1}{2\gamma^2}$$

More Generally…

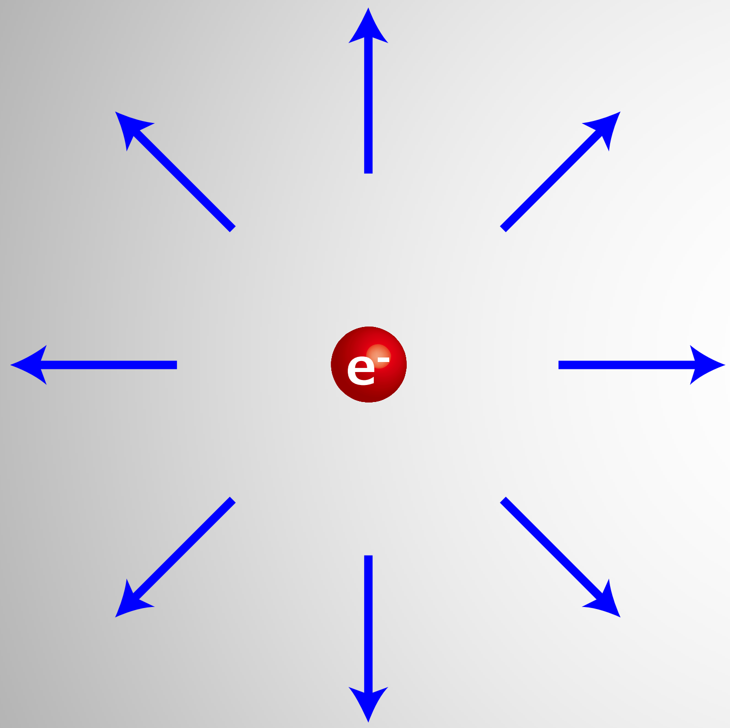


$$
\begin{aligned}
R_2 &= \sqrt{(R_1^2 + (v\Delta t)^2 - 2R_1 v\Delta t\cos\theta)} \\
&\sim R_1 - (\boldsymbol{v}\cdot\boldsymbol{n})\Delta t \\
\Delta\tau &= t_2 - t_1 = \Delta t + R_2/c - R_1/c \\
&= \Delta t\,(1 - \boldsymbol{\beta}\cdot\boldsymbol{n}) = \Delta t\left(\frac{1}{2\gamma^2} + \theta^2\right)
\end{aligned}
$$

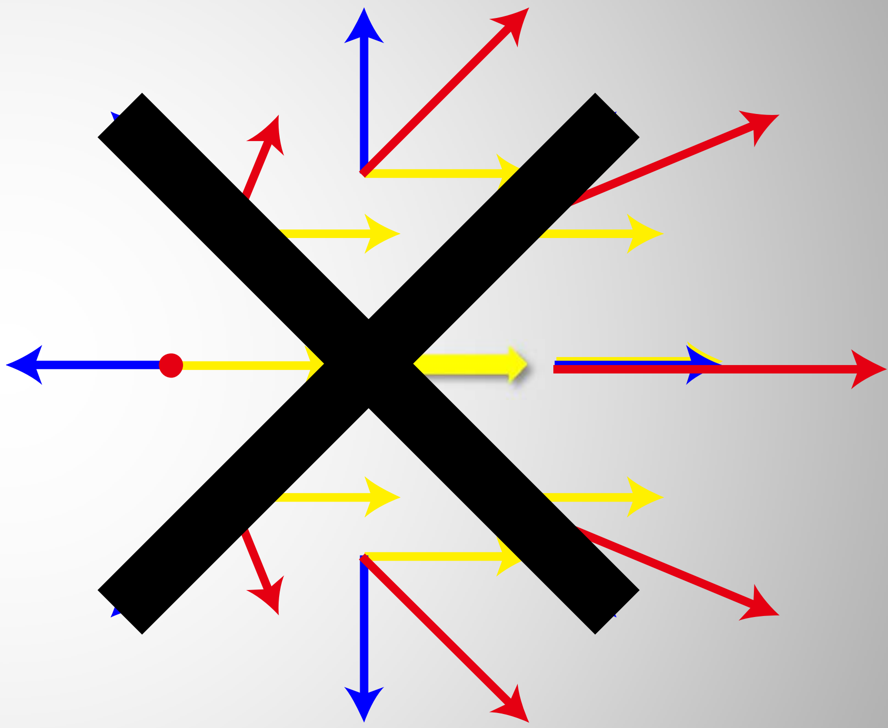
$\gamma >> 1, \theta \sim 0$

Time Squeezing

Photons Emitted by a Moving Electron

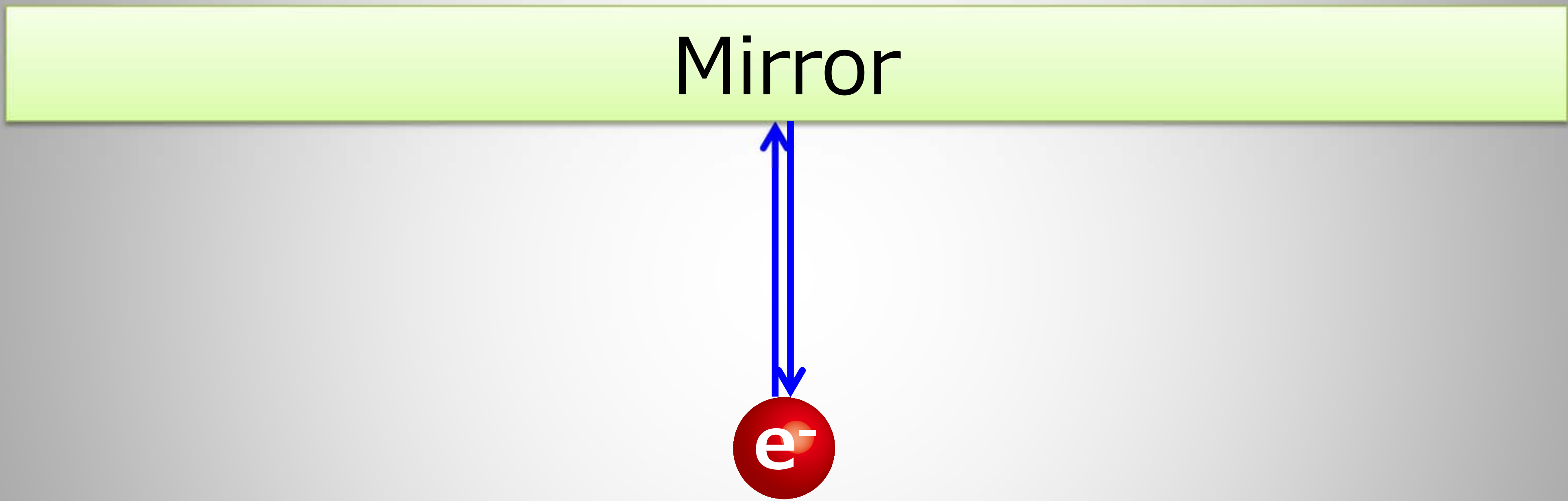


Rest Electron



Moving Electron ($v \sim c$)

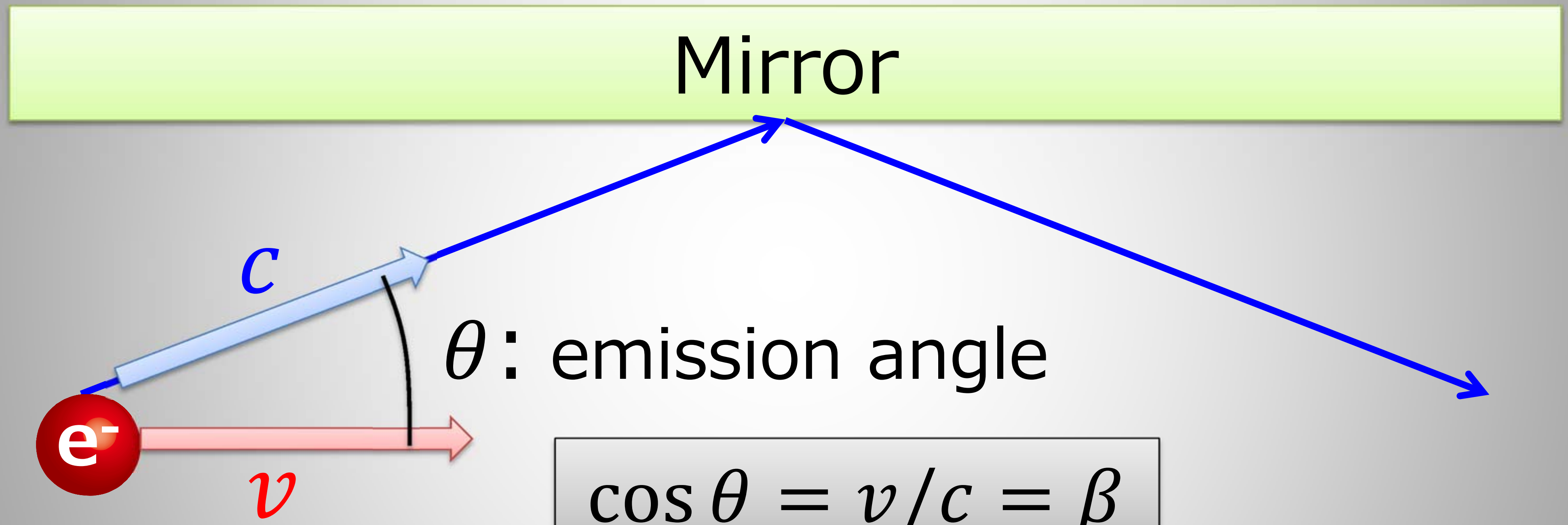
If Photons Are Reflected by a Mirror?



Rest Frame of e^-

If Photons Are Reflected by a Mirror?

Laboratory Frame



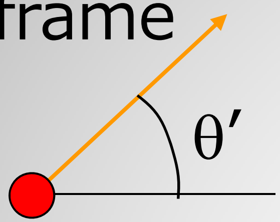
$$\boxed{\cos\theta = v/c = \beta}$$

✓ $\cos\theta \sim 1 - \theta^2/2$
✓ $\beta \sim 1 - \gamma^{-2}/2$

$$\boxed{\theta \sim \gamma^{-1} \ll 1}$$

More Generally…

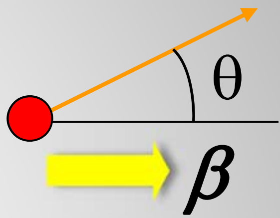
rest frame

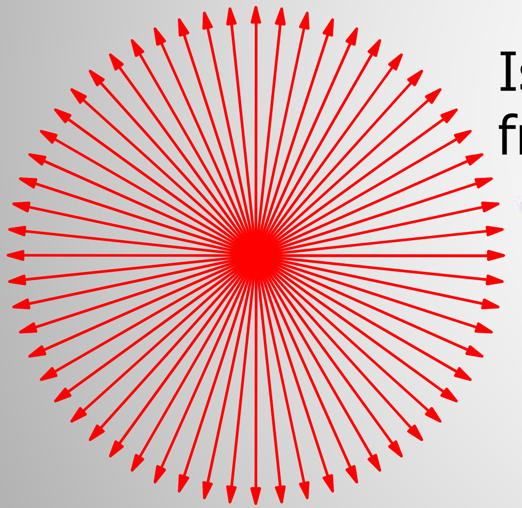


conversion equation

$$\theta = \tan^{-1}\left(\frac{\gamma^{-1}\sin\theta'}{\beta + \cos\theta'}\right)$$

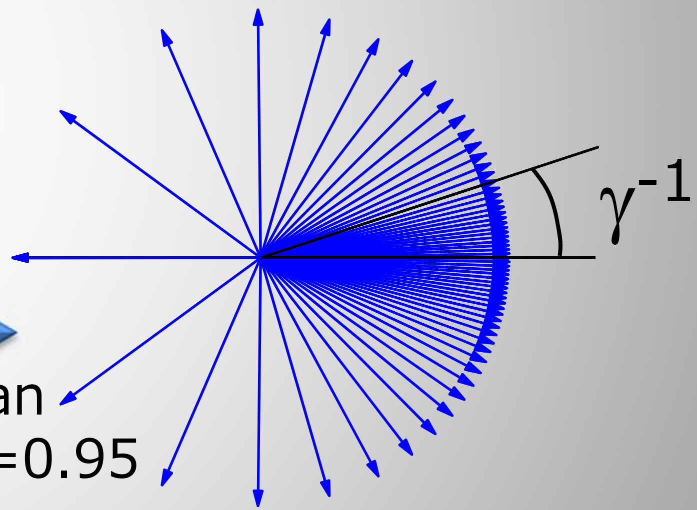
lab. frame





Isotropic emission
from a rest electron

Emission from an
electron with β=0.95



Light emitted from a moving object (β~1) concentrates within γ^{-1}

SR from a High-Energy Electron

High Energy Electron

$$\beta \sim 1 - \frac{1}{2\gamma^2}$$

Large Time Squeezing

Short Wavelength

$$\theta \leq \gamma^{-1} \ll 1$$

Radiation in the Forward Direction

High Directivity