Coherence: from Undulators to Free Electron Lasers

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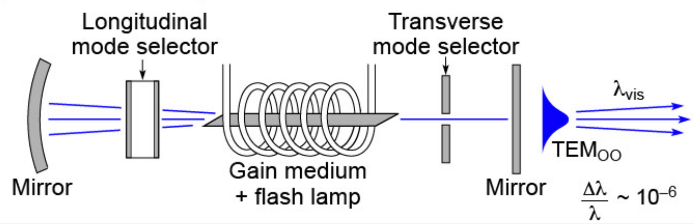
SPring-8

Coherence

Laser Cavity



Spatial and Temporal Coherence

$$d \cdot 2\theta \big|_{FWHM} \simeq \frac{\lambda}{2}$$

$$\ell_{coh} = \lambda^2/2\Delta\lambda$$

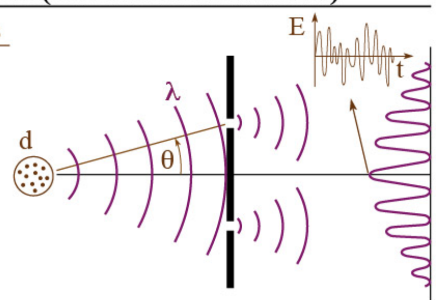
$$\tau_{coh} = \ell_{coh}/c$$

$$\Delta E \cdot \Delta\tau \big|_{FWHM} \geq 1.82\ \text{eV} \cdot \text{fsec}$$

Young's Double Slit (uncorrelated emitters)

N uncorrelated emitters

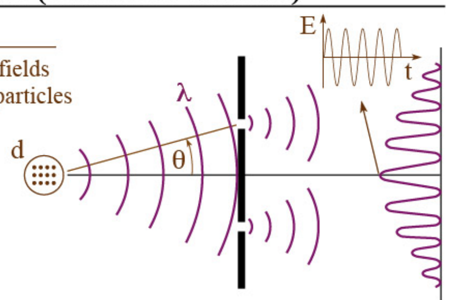
- Self-interference only
- Electric fields chaotic
- Intensities add
- Radiated power ~ N



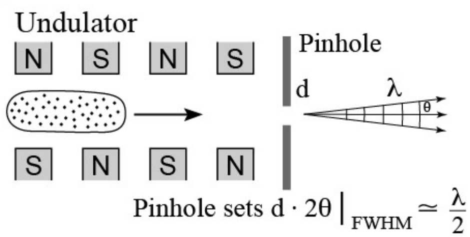
Young's Double Slit (correlated emitters)

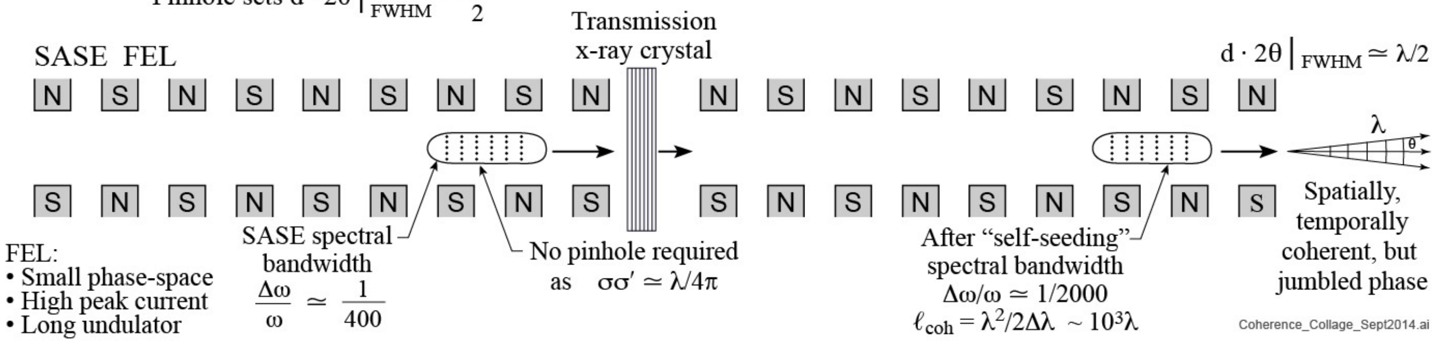
N correlated emitters

- Phase coherent electric fields
- Electric fields from all particles interfere constructively
- Radiated power ~ N^2



Undulators and Free-Electron Laser





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Coherence

co·here \kō-ˈhi(ə)r\ *vi* [L *cohaerēre*, fr. *co-* + *haerēre* to stick] **1 a :** to hold together firmly as parts of the same mass **b :** ADHERE **c :** to display cohesion **2 :** to consist of parts that cohere **3 a :** to become united in principles, relationships, or interests **b :** to be logically or aesthetically consistent **syn** see STICK

co·her·ence \kō-ˈhir-ən(t)s, -ˈher-\ *n* **:** the quality or state of cohering; *esp* **:** systematic connection esp. in logical discourse

co·her·en·cy \-ən-sē\ *n* **:** COHERENCE

co·her·ent \kō-ˈhir-ənt, -ˈher-\ *adj* [MF or L; MF *cohérent*, fr. L *cohaerent-*, *cohaerens*, prp. of *cohaerēre*] **1 :** having the quality of cohering **2 :** logically consistent **3 :** having waves in phase and of one wavelength ⟨~ light⟩ — **co·her·ent·ly** *adv*

Webster's 7th Collegiate Dictionary (1971)

co·her·ence (kō-hîrʹəns, -hĕrʹ-) also **co·her·en·cy** (-ən-sē) *n.* The quality or state of cohering, esp. logical or orderly relationship of parts.

co·her·ent (kō-hîrʹənt, -hĕrʹ-) *adj.* **1.** Sticking together; cohering. **2.** Marked by an orderly or logical relation of parts that affords comprehension or recognition: *coherent speech.* **3.** *Physics.* Of or pertaining to waves with a continuous relationship among phases. **4.** Of or pertaining to a system of units of measurement in which a small number of basic

American Heritage 2nd College Edition Dictionary (1985)

- Coherence (physics), an ideal property of waves that enables stationary (i.e. temporally and spatially constant) interference
- Coherence time, the time over which a propagating wave (especially a laser or maser beam) may be considered coherent; the time interval within which its phase is, on average, predictable

Wikipedia, the free encyclopedia

Born and Wolf, Chapter 10

Interference and diffraction with partially coherent light

10.1 Introduction

So far we have been mainly concerned with monochromatic light produced by a point source. Light from a real physical source is never strictly monochromatic, since even the sharpest spectral line has a finite width. Moreover, a physical source is not a point source, but has a finite extension, consisting of very many elementary radiators (atoms). The disturbance produced by such a source may be expressed, according to Fourier's theorem, as the sum of strictly monochromatic and therefore infinitely long wave trains. The elementary monochromatic theory is essentially concerned with a single component of this Fourier representation.

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Consider next the light disturbances at two points P_1 and P_2 in a wave field produced by an extended quasi-monochromatic source. For simplicity assume that the wave field is in vacuum and that P_1 and P_2 are many wavelengths away from the source. We may expect that, when P_1 and P_2 are close enough to each other, the fluctuations of the amplitudes at these points, and also the fluctuations of the phases, will not be independent. It is reasonable to suppose that, if P_1 and P_2 are so close to each other that the difference $\Delta S = SP_1 - SP_2$ between the paths from each source point S is small compared to the mean wavelength $\bar{\lambda}$, then the fluctuations at P_1 and P_2 will effectively be the same; and that some correlation between the fluctuations will exist even for greater separations of P_1 and P_2, provided that for all source points the path difference ΔS does not exceed the coherence length $c\Delta t \sim c/\Delta\nu = \bar{\lambda}^2/\Delta\lambda$. We are thus led to the concept of a *region of coherence* around any point P in a wave field.

In order to describe adequately a wave field produced by a finite polychromatic source it is evidently desirable to introduce some measure for the correlation that exists between the vibrations at different points P_1 and P_2 in the field. We must expect such a measure to be closely related to the sharpness of the interference fringes which would result on combining the vibrations from the two points. We must expect sharp fringes when the correlation is high (e.g. when the light at P_1 and P_2 comes from a very small source of a narrow spectral range), and no fringes at all in the absence of correlation (e.g. when P_1 and P_2 each receive light from a different physical source). We described these situations by the terms 'coherent' and 'incoherent' respectively. In general neither of these situations is realized and we may speak of vibrations which are *partially coherent*.

Physical sources are of finite size and duration

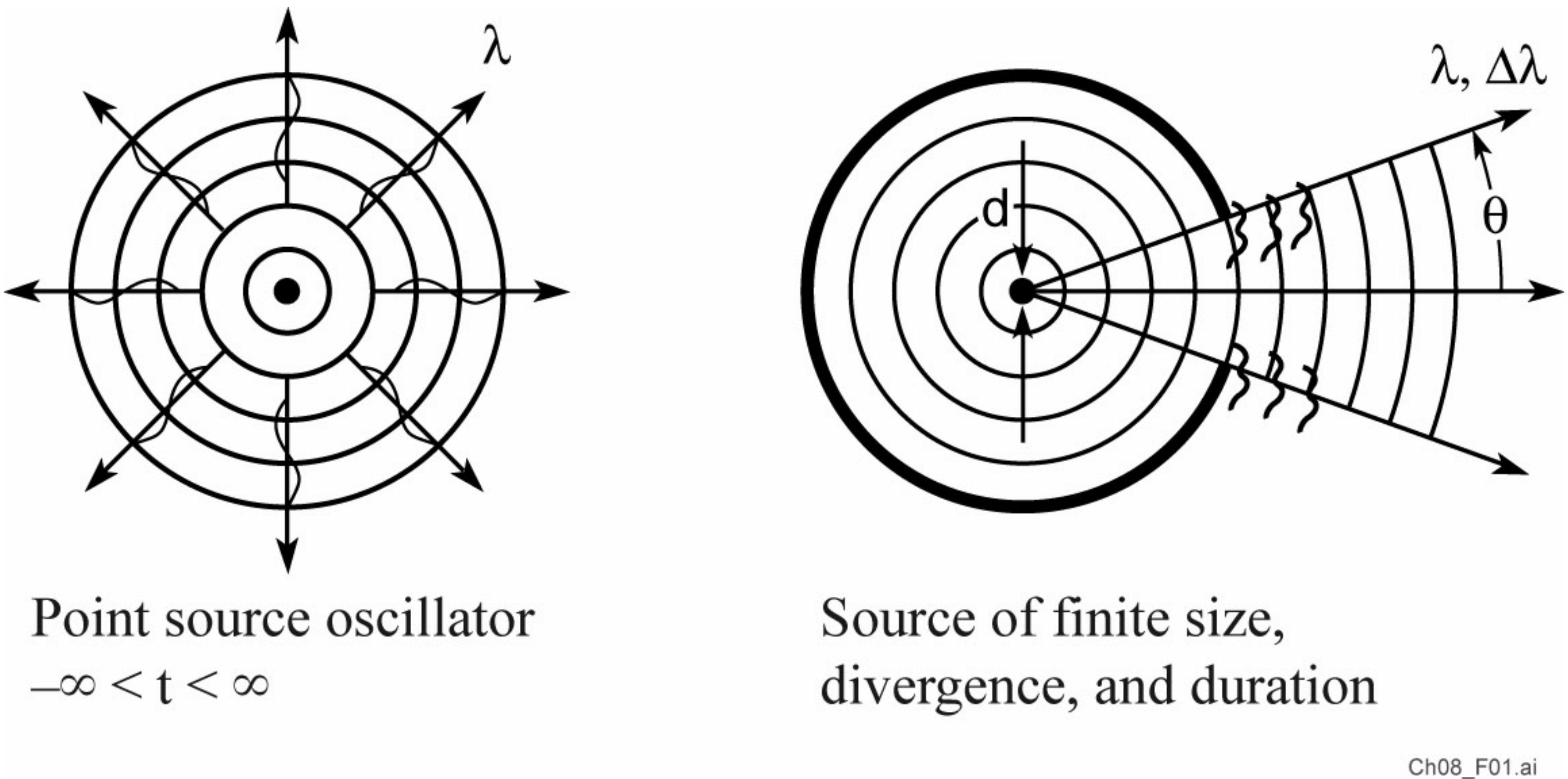
Regions of coherence
$\ell_{coh} = c\Delta t = \lambda^2/2\Delta\lambda$

Normalized degree of coherence

$$\gamma_{12}(\tau) \equiv \frac{\langle E_1(t+\tau)E_2^*(t)\rangle}{\sqrt{\langle |E_1|^2\rangle}\sqrt{\langle |E_2|^2\rangle}}$$

Coherent
Incoherent
Partially incoherent

Coherence, partial coherence, and incoherence



Spatial and temporal coherence

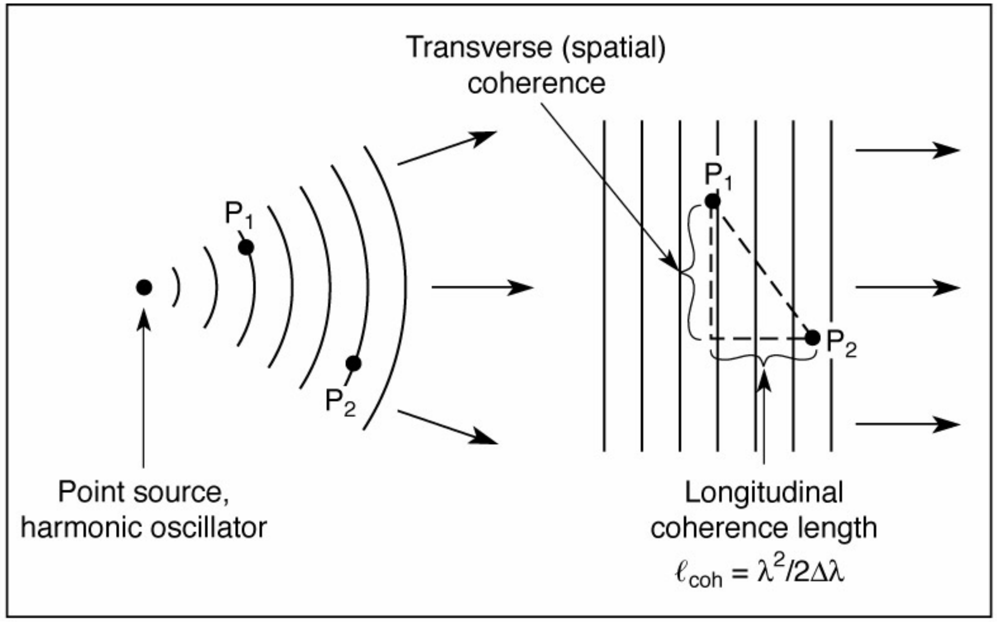
Mutual coherence factor

$$\Gamma_{12}(\tau) \equiv \langle E_1(t+\tau)E_2^*(t)\rangle \qquad (8.1)$$

Normalize degree of spatial coherence (complex coherence factor)

$$\mu_{12} = \frac{\langle E_1(t)E_2^*(t)\rangle}{\sqrt{\langle |E_1|^2\rangle}\sqrt{\langle |E_2|^2\rangle}} \qquad (8.12)$$

A high degree of coherence ($\mu \to 1$) implies an ability to form a high contrast interference (fringe) pattern. A low degree of coherence ($\mu \to 0$) implies an absence of interference, except with great care. In general radiation is partially coherent.



Longitudinal (temporal) coherence length

$$\ell_{coh} = \frac{\lambda^2}{2\,\Delta\lambda} \qquad (8.3)$$

Full spatial (transverse) coherence

$$d \cdot \theta = \lambda/2\pi \qquad (8.5)$$

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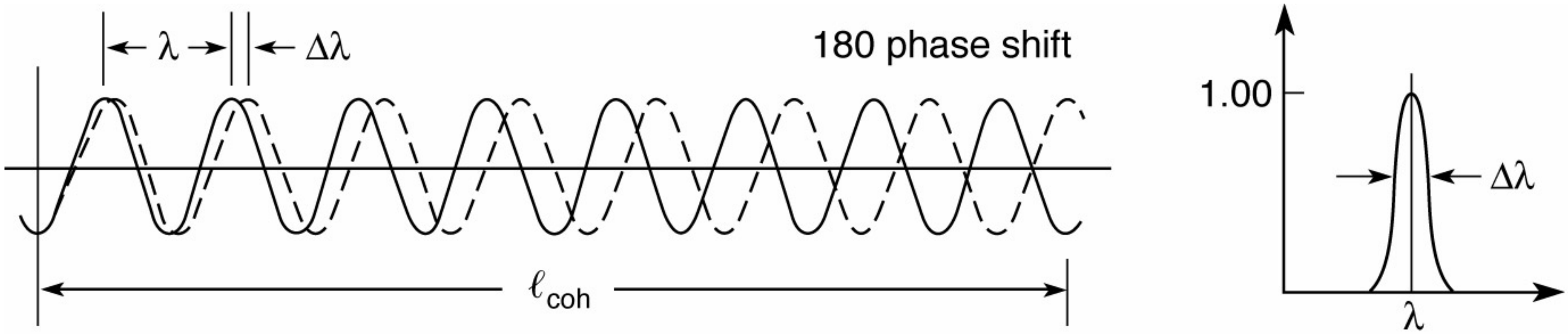
Marching band and coherence lengths

Spectral bandwidth and longitudinal coherence length



Define a coherence length ℓ_{coh} as the distance of propagation over which radiation of spectral width $\Delta\lambda$ becomes 180° out of phase. For a wavelength λ propagating through N cycles

$$\ell_{coh} = N\lambda$$

and for a wavelength $\lambda + \Delta\lambda$, a half cycle less $(N - \frac{1}{2})$

$$\ell_{coh} = (N - \tfrac{1}{2})\,(\lambda + \Delta\lambda)$$

Equating the two

$$N = \lambda/2\Delta\lambda$$

so that

$$\boxed{\ell_{coh} = \frac{\lambda^2}{2\,\Delta\lambda}} \qquad (8.3)$$

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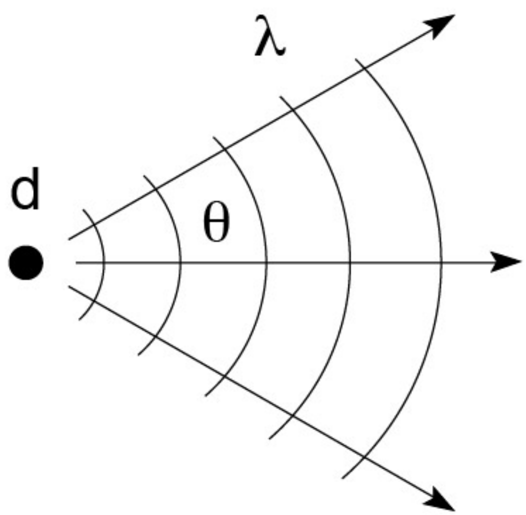
A practical interpretation of spatial coherence

- **Associate spatial coherence with a spherical wavefront.**
- **A spherical wavefront implies a point source.**
- **How small is a "point source"?**



From Heisenberg's Uncertainty Principle ($\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$), the smallest source size "d" you can resolve, with wavelength λ and half angle θ, is

$$\mathbf{d \cdot \theta} = \frac{\lambda}{2\pi} \qquad \text{(rms)}$$

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Partially coherent radiation approaches uncertainty principle limits

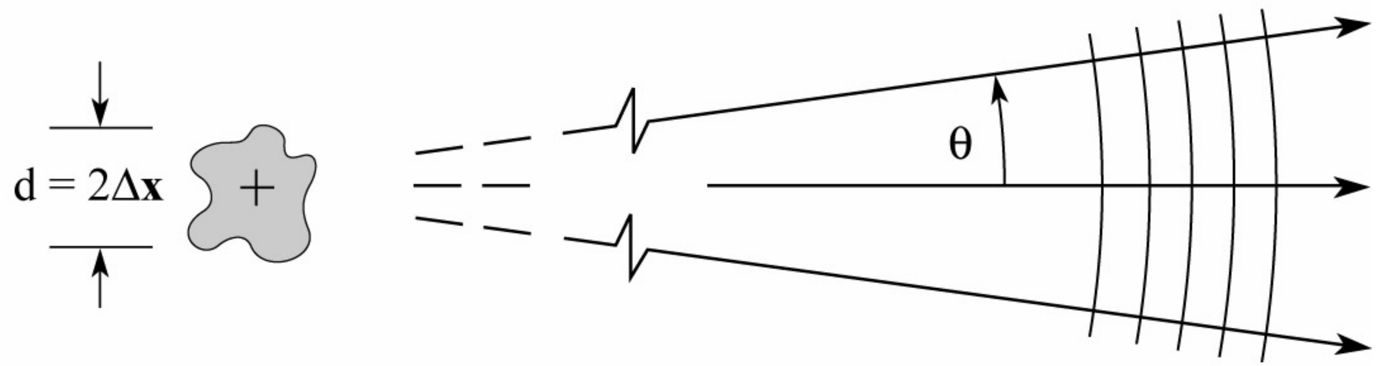
$\Delta \mathbf{x} \cdot \Delta \mathbf{p} \geq \hbar/2$ (8.4)

Standard deviations of Gaussian distributed functions (Tipler, 1978, pp. 174-189)

$\Delta \mathbf{x} \cdot \hbar \Delta \mathbf{k} \geq \hbar/2$

$\Delta \mathbf{x} \cdot \mathbf{k} \Delta \theta \geq 1/2$

$2\Delta \mathbf{x} \cdot \Delta \theta \geq \lambda/2\pi$

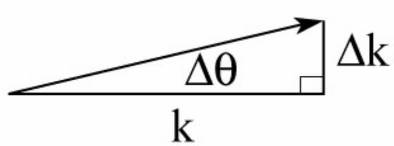


Spherical wavefronts occur in the limiting case

Note:

$\Delta \mathbf{p} = \hbar \Delta k$

$\Delta k = k \Delta \theta$



$$\boxed{d \cdot \theta = \lambda/2\pi}$$

(spatially coherent) $\Big\}$ $\frac{1}{\sqrt{e}}$ quantities

or

$$(d \cdot 2\theta)_{FWHM} \simeq \lambda/2$$ $\}$ FWHM quantities

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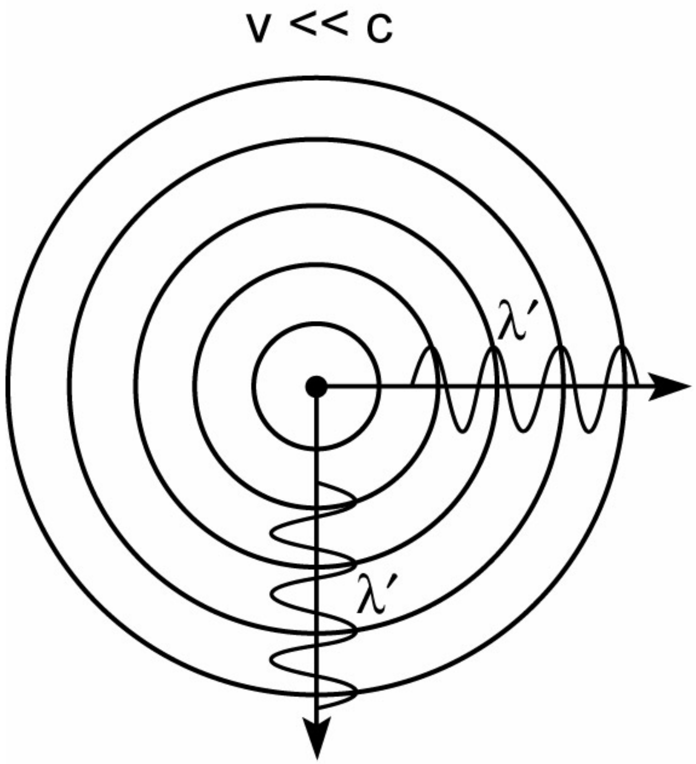
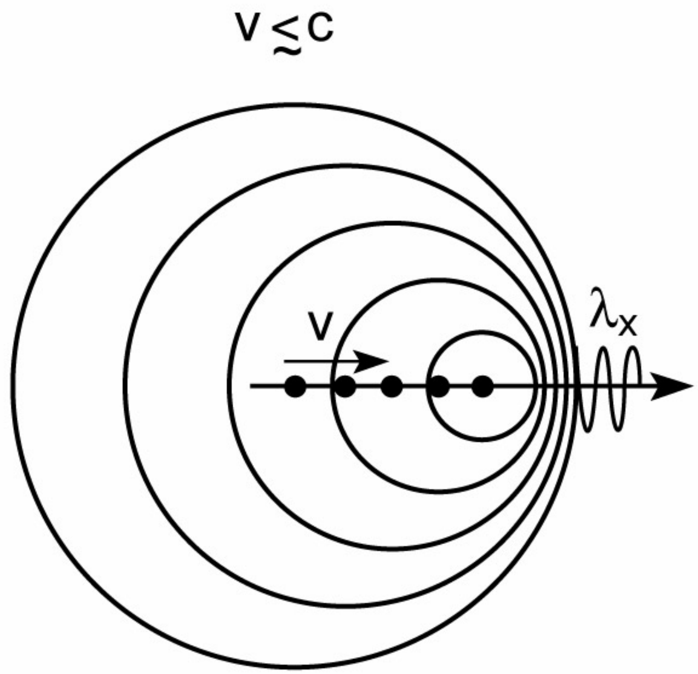
X-rays from relativistic electrons





Note: Angle-dependent doppler shift

$$\lambda = \lambda' \left(1 - \frac{v}{c}\cos\theta\right) \qquad \lambda = \lambda' \gamma \left(1 - \frac{v}{c}\cos\theta\right)$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Courtesy of John Madey

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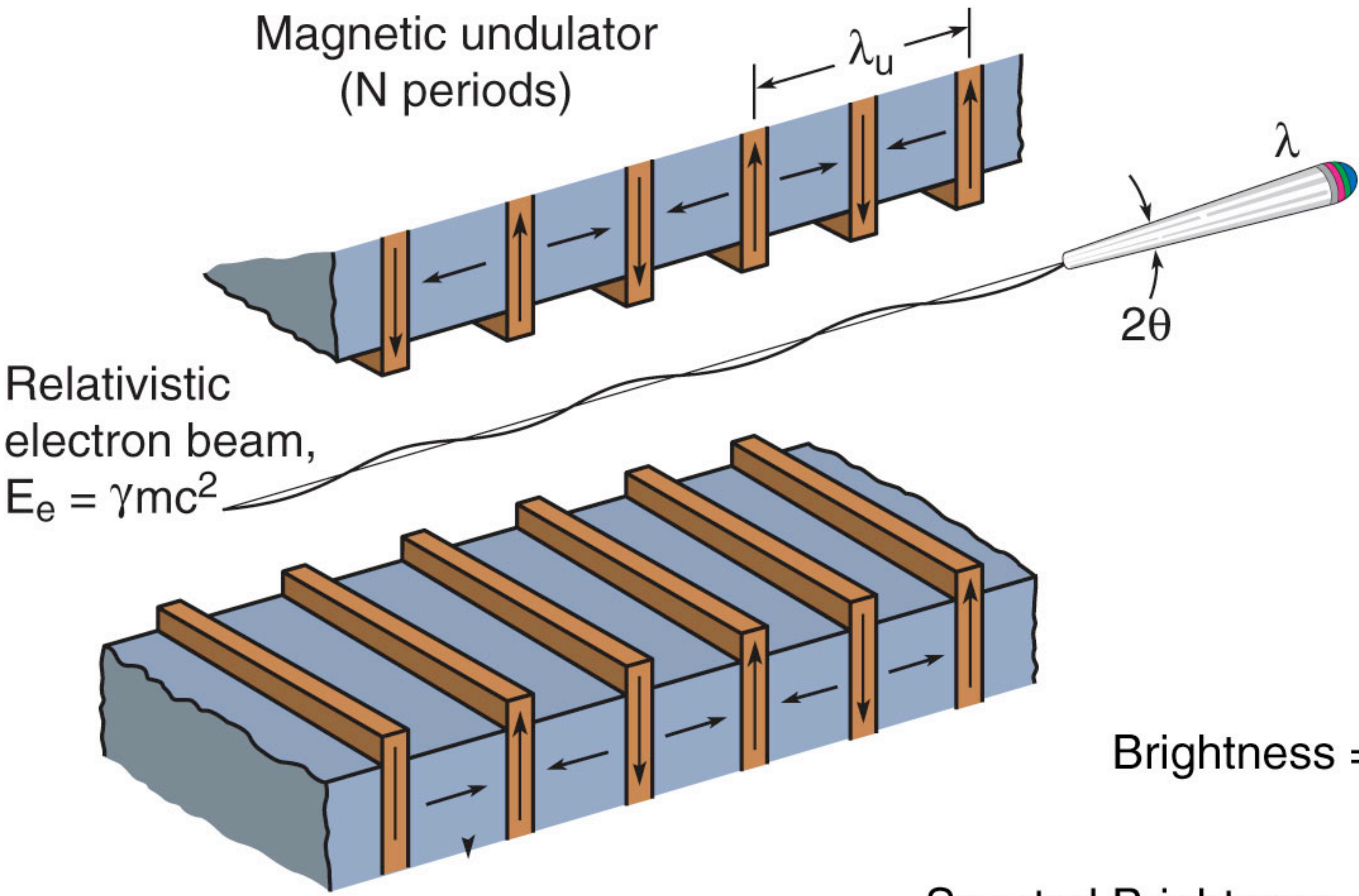
Undulator radiation from a small electron beam radiating into a narrow forward cone is very bright



$$\lambda \simeq \frac{\lambda_u}{2\gamma^2}$$

$$\theta_{cen} \simeq \frac{1}{\gamma\sqrt{N}}$$

$$\left[\frac{\Delta\lambda}{\lambda}\right]_{cen} = \frac{1}{N}$$

$$\text{Brightness} = \frac{\text{photon flux}}{(\Delta A)\,(\Delta\Omega)}$$

$$\text{Spectral Brightness} = \frac{\text{photon flux}}{(\Delta A)\,(\Delta\Omega)\,(\Delta\lambda/\lambda)}$$

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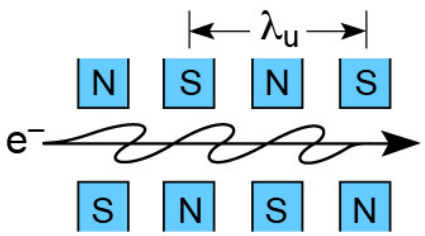
Undulator radiation

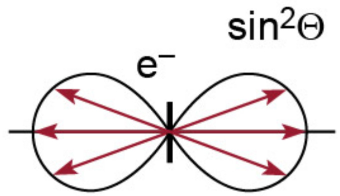
Laboratory Frame of Reference	Frame of Moving e⁻	Frame of Observer	Following Monochromator



$E = \gamma mc^2$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

N = # periods

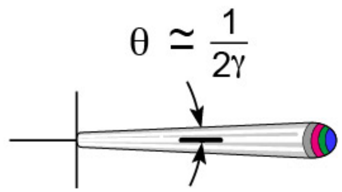


e^- radiates at the Lorentz contracted wavelength:

$$\lambda' = \frac{\lambda_u}{\gamma}$$

Bandwidth:

$$\frac{\lambda'}{\Delta\lambda'} \simeq N$$



Doppler shortened wavelength on axis:

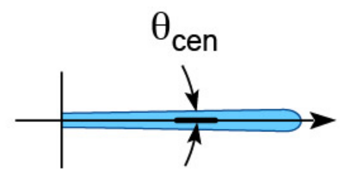
$$\lambda = \lambda'\gamma(1 - \beta\cos\theta)$$

$$\lambda = \frac{\lambda_u}{2\gamma^2}(1 + \gamma^2\theta^2)$$

Accounting for transverse motion due to the periodic magnetic field:

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)$$

where $K = eB_0\lambda_u/2\pi mc$



For $\frac{\Delta\lambda}{\lambda} \simeq \frac{1}{N}$

$$\theta_{cen} \simeq \frac{1}{\gamma\sqrt{N}}$$

typically

$\theta_{cen} \simeq$ 10-30 μrad

Following Albert Hoffman

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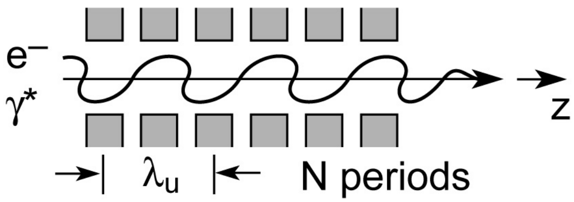
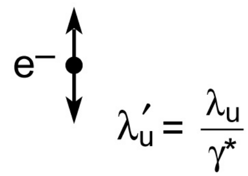
Calculating Power in the Central Radiation Cone: Using the well known "dipole radiation" formula by transforming to the frame of reference moving with the electrons

x, z, t laboratory frame of reference



Lorentz transformation

x′, z′, t′ frame of reference moving with the average velocity of the electron

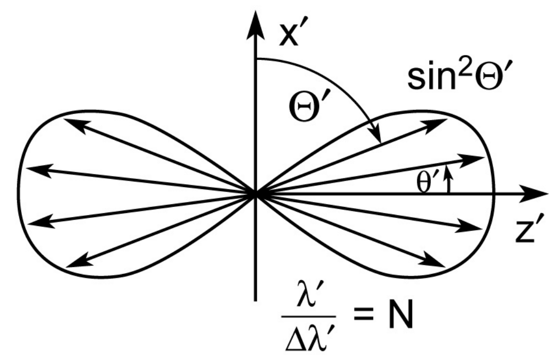


Determine *x, z, t* motion:

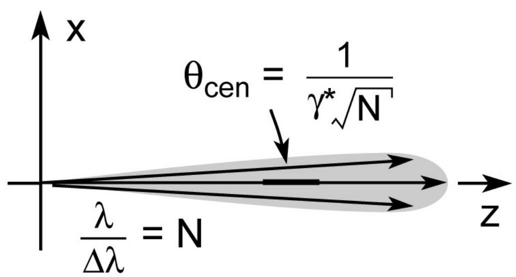
$$\frac{d\mathbf{p}}{dt} = -e\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Dipole radiation:

$$\frac{dP'}{d\Omega'} = \frac{e^2 a'^2 \sin^2 \Theta'}{16\pi^2 \epsilon_0 c^3}$$



Lorentz transformation



$$\bar{P}_{cen} = \frac{\pi e \gamma^2 I}{\epsilon_0 \lambda_u} \frac{K^2}{(1 + K^2/2)^2}$$

$$\frac{dP'}{d\Omega'} = \frac{e^2 c \gamma^2}{4\epsilon_0 \lambda_u^2} \frac{K^2}{(1 + K^2/2)^2} (1 - \sin^2 \theta' \cos^2 \phi') \cos^2 \omega'_u t'$$

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Power in the central cone

$$\lambda_x = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)$$

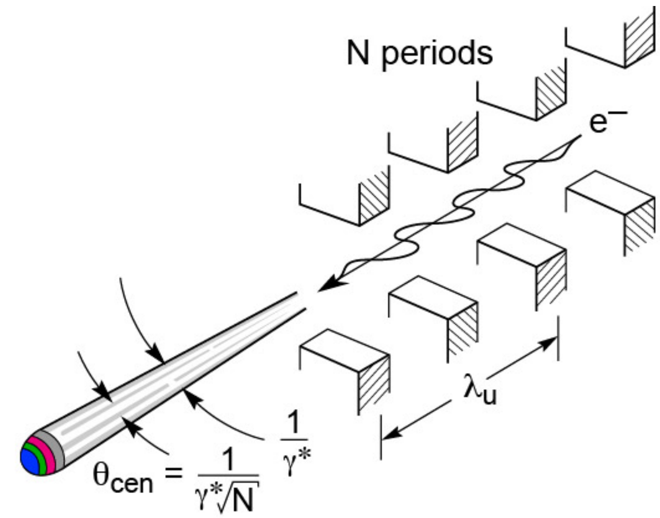
$$\bar{P}_{cen} = \frac{\pi e \gamma^2 I}{\epsilon_0 \lambda_u} \frac{K^2}{(1 + \frac{K^2}{2})^2} [JJ]^2$$

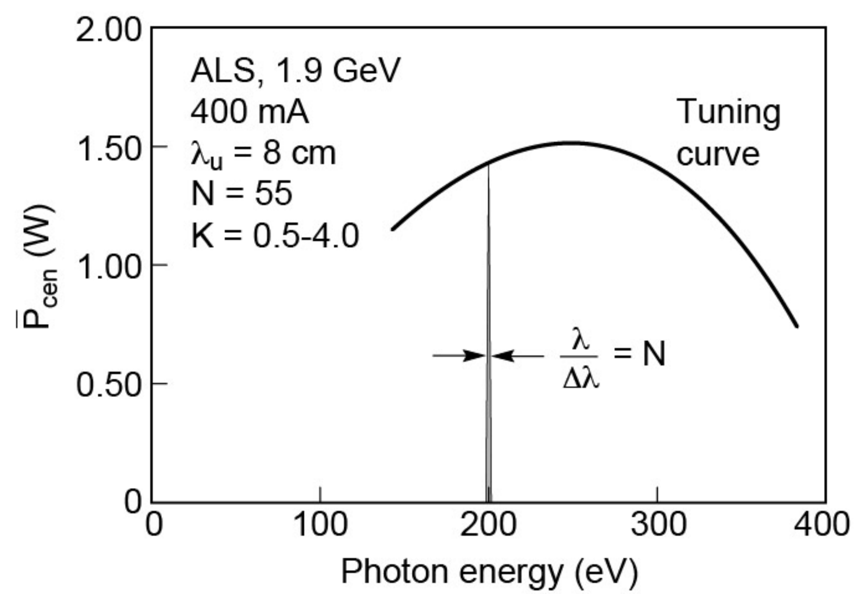
$$\theta_{cen} = \frac{1}{\gamma^* \sqrt{N}}$$

$$\left(\frac{\Delta\lambda}{\lambda}\right)_{cen} = \frac{1}{N}$$

$$K = \frac{eB_0\lambda_u}{2\pi m_0 c}$$

$$\gamma^* = \gamma \Big/ \sqrt{1 + \frac{K^2}{2}}$$





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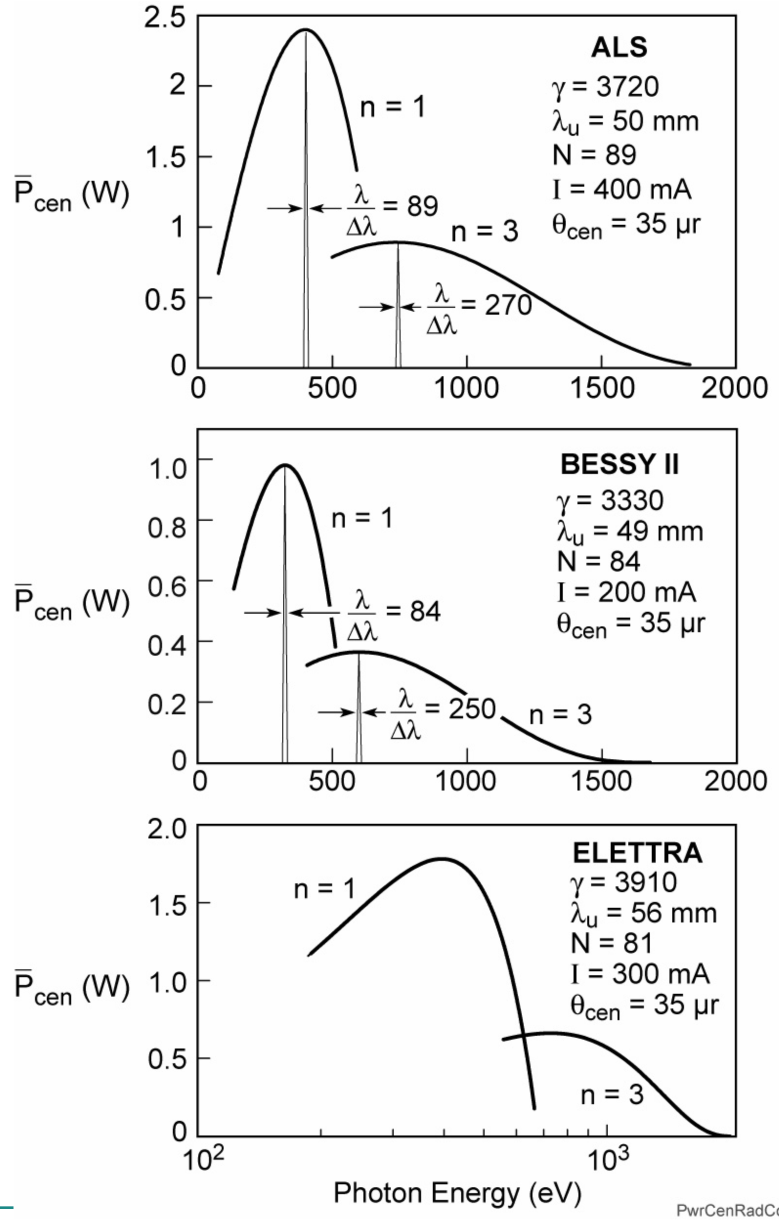
Power in the central radiation cone for three soft x-ray undulators

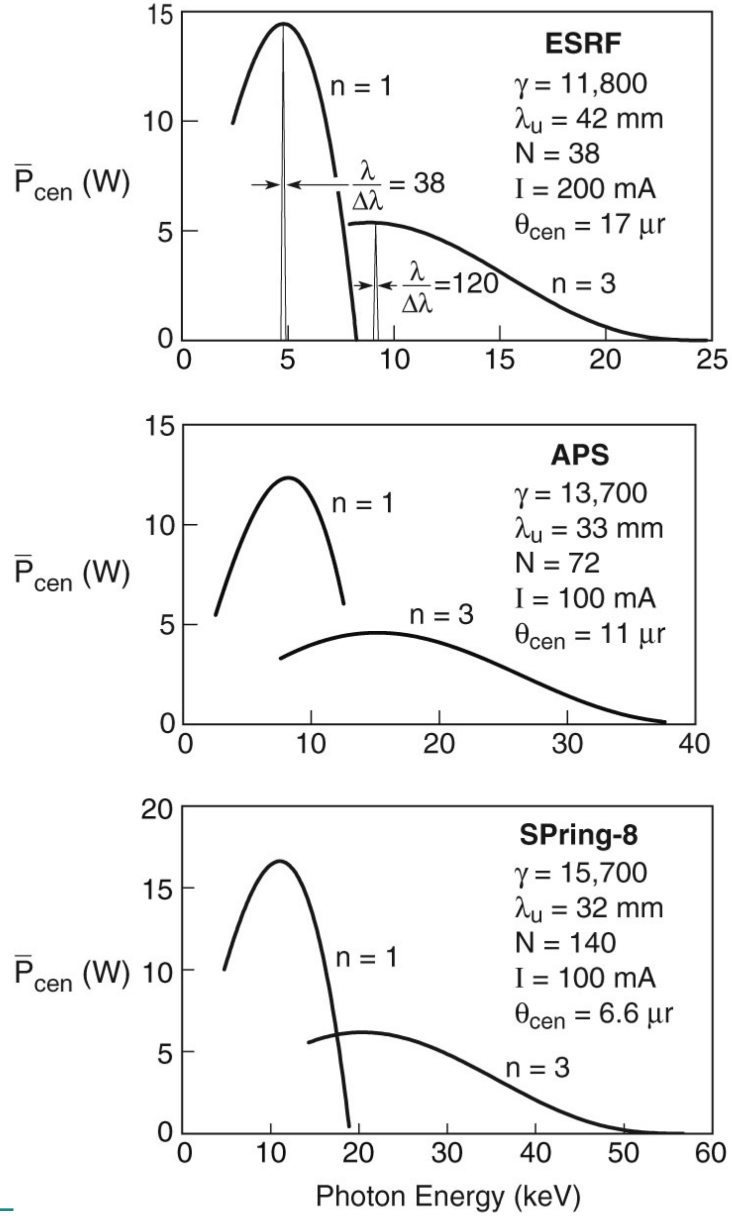


$$\theta_{cen} = \frac{1}{\gamma^* \sqrt{N}}$$

$$\left[\frac{\Delta\lambda}{\lambda}\right]_1 = \frac{1}{N}$$

$$\left[\frac{\Delta\lambda}{\lambda}\right]_3 = \frac{1}{3N}$$

Power in the central radiation cone for three hard x-ray undulators



$$\theta_{cen} = \frac{1}{\gamma^* \sqrt{N}}$$

$$\left[\frac{\Delta\lambda}{\lambda}\right]_1 = \frac{1}{N}$$

$$\left[\frac{\Delta\lambda}{\lambda}\right]_3 = \frac{1}{3N}$$

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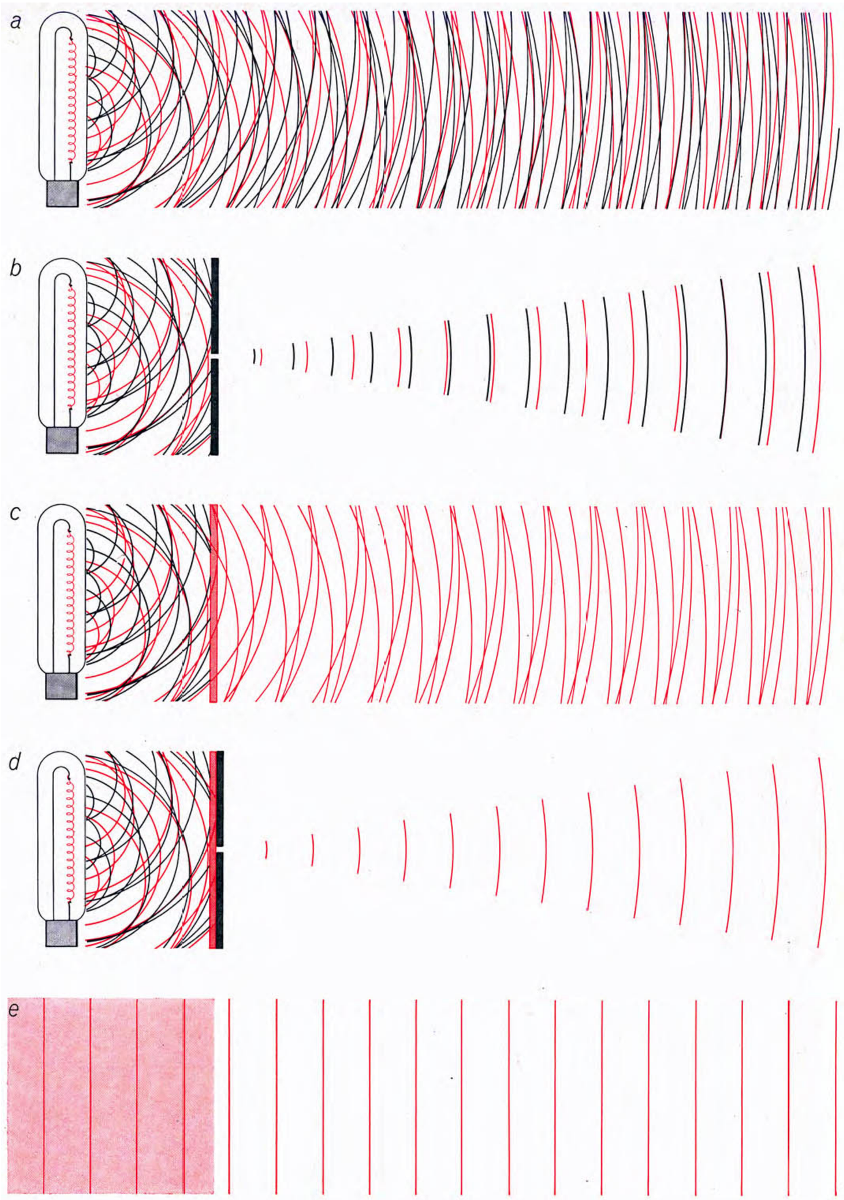
Ordinary light and laser light



Ordinary thermal light source, atoms radiate independently.

A pinhole can be used to obtain spatially coherent light, but at a great loss of power.

A color filter (or monochromator) can be used to obtain temporally coherent light, also at a great loss of power.

Pinhole and spectral filtering can be used to obtain light which is both spatially and temporally coherent but the power will be very small (tiny).

All of the laser light is both spatially and temporally coherent*.

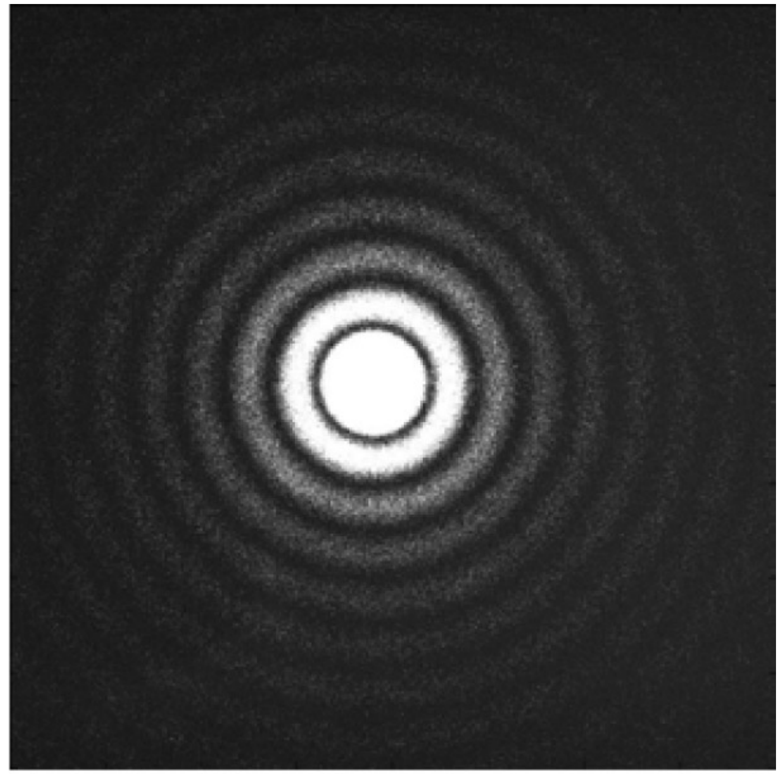
Arthur Schawlow, “Laser Light”, Sci. Amer. 219, 120 (Sept. 1968)

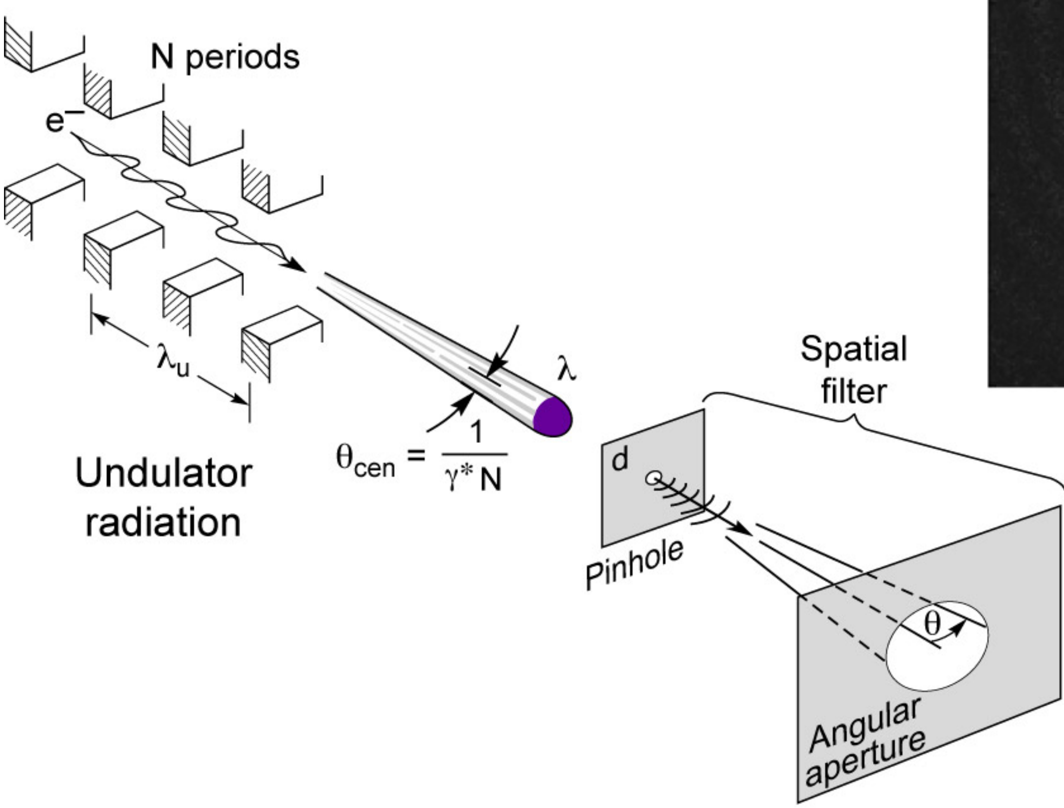
Spatially coherent undulator radiation

$l_{coh} = \lambda^2/2\Delta\lambda$ {temporal (longitudinal) coherence} (8.3)

$d \cdot \theta = \lambda/2\pi$ {spatial (transverse) coherence} (8.5)

or $d \cdot 2\theta|_{FWHM} = 0.44\,\lambda$ (8.5*)





λ = 2.48 nm (600 eV)
d = 2.5 μm
t = 200 msec
ALS beamline 12.0.2
λ_u = 80 mm, N = 55, n = 3
25 mm wide CCD at 410 mm

Courtesy of Kris Rosfjord, UCB

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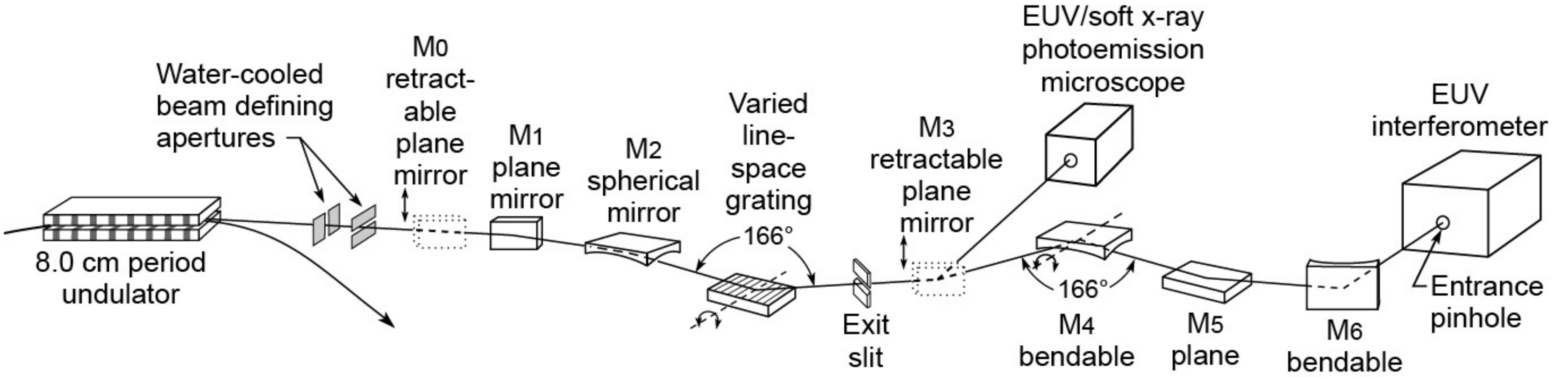
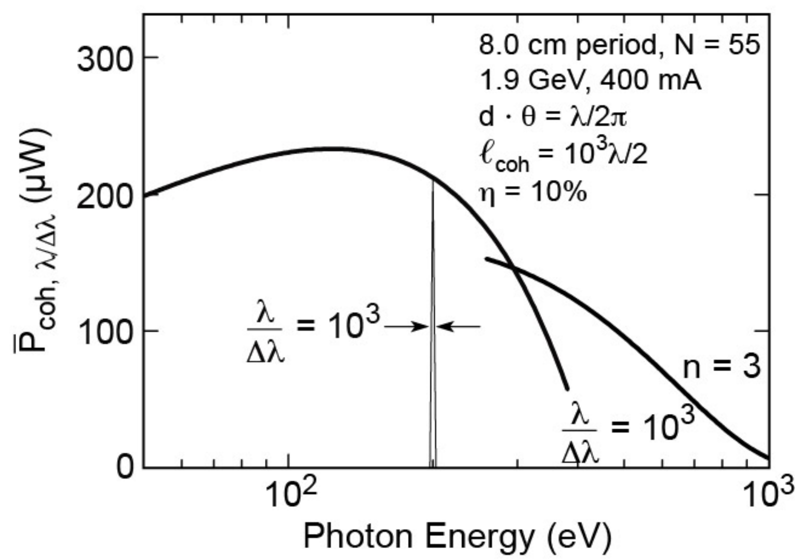
Spatially and spectrally filtered undulator radiation

- Pinhole filtering for full spatial coherence
- Monochromator for spectral filtering to $\lambda/\Delta\lambda > N$



$$\bar{P}_{\mathrm{coh},\lambda/\Delta\lambda} = \underbrace{\eta}_{\text{beamline efficiency}} \underbrace{\frac{(\lambda/2\pi)^2}{(d_x\theta_x)(d_y\theta_y)}}_{\text{spatial filtering}} \cdot \underbrace{N\frac{\Delta\lambda}{\lambda}}_{\text{spectral filtering}} \cdot \bar{P}_{\mathrm{cen}} \qquad (8.10a)$$



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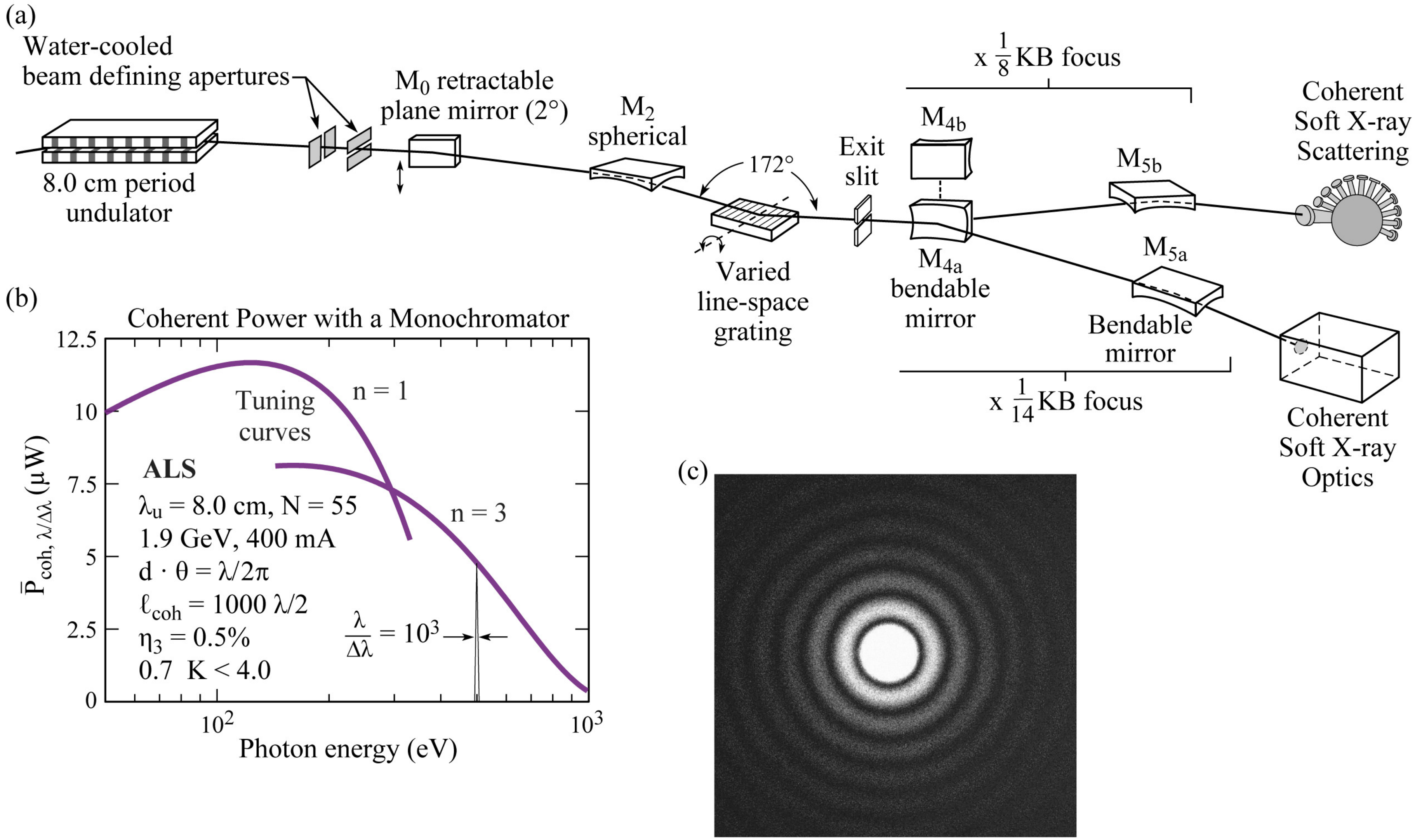
A soft x-ray coherence beamline at the ALS



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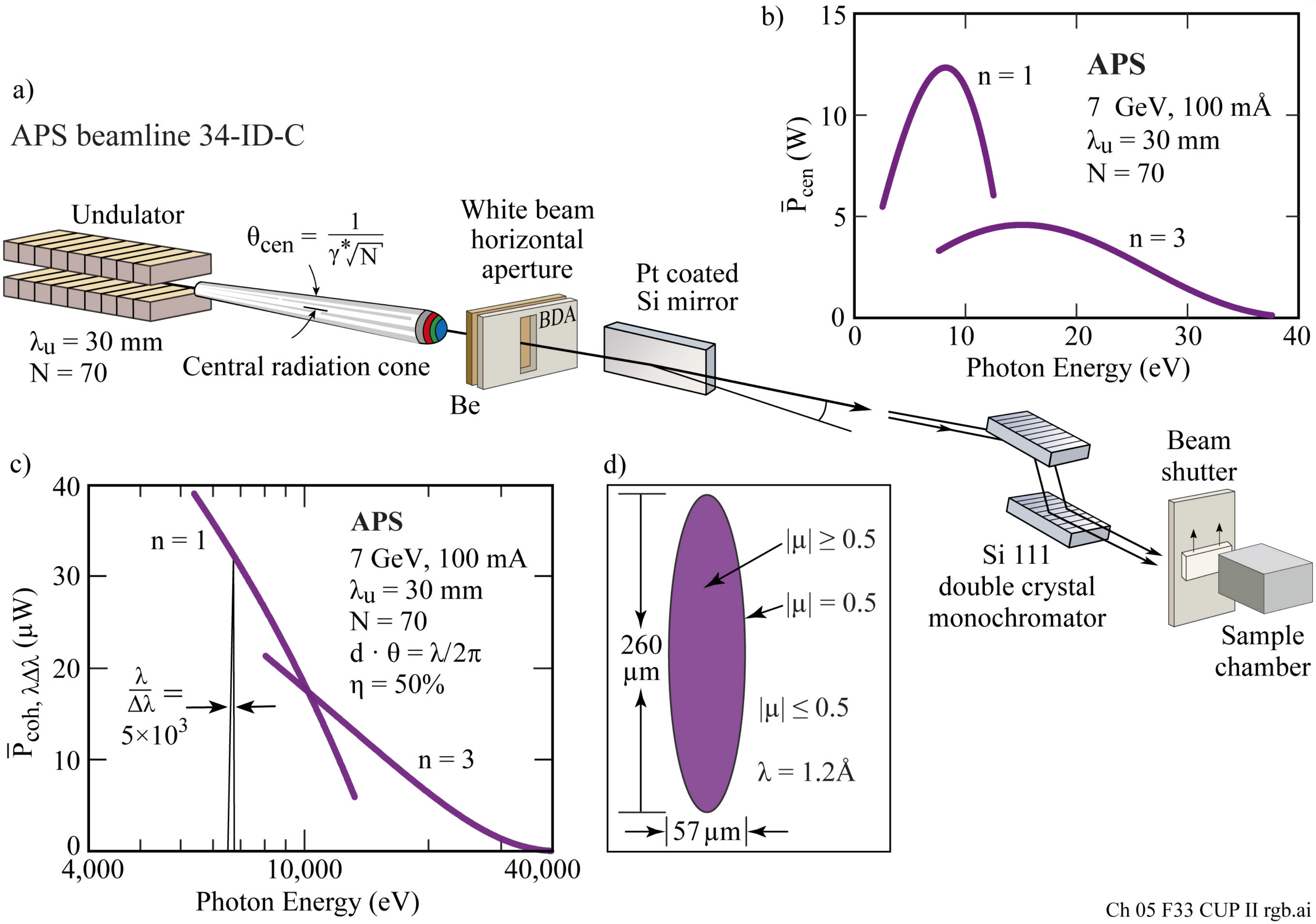
A hard x-ray coherence beamline at the APS



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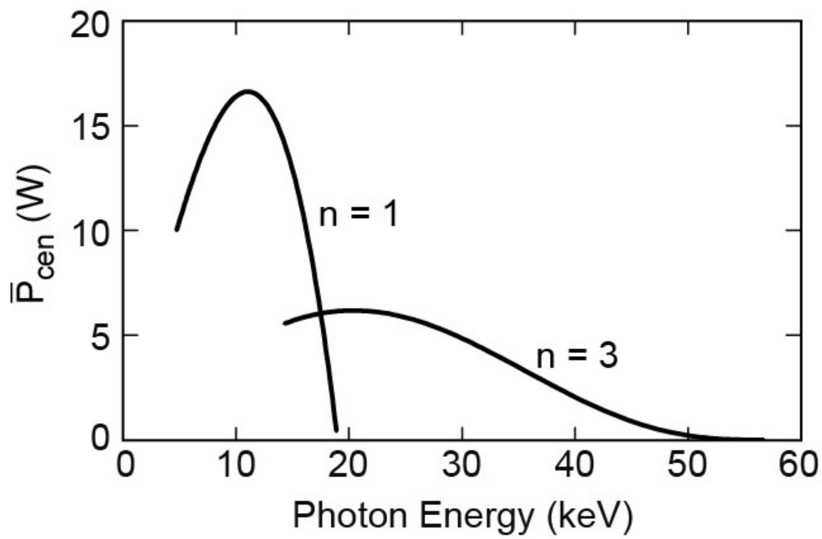
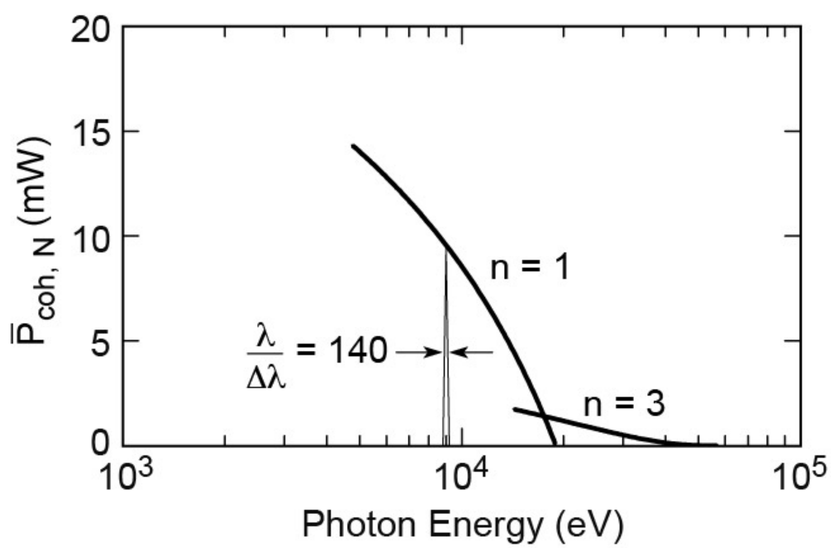
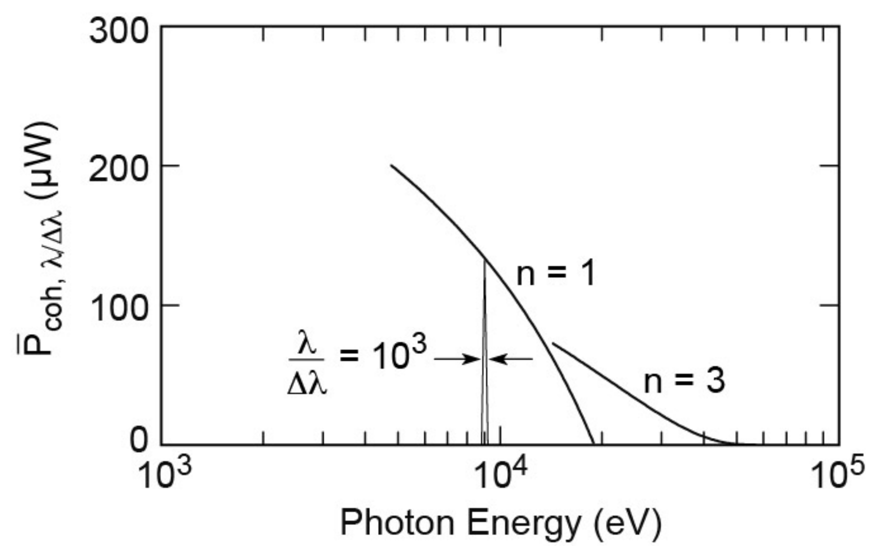
Coherent power at SPring-8



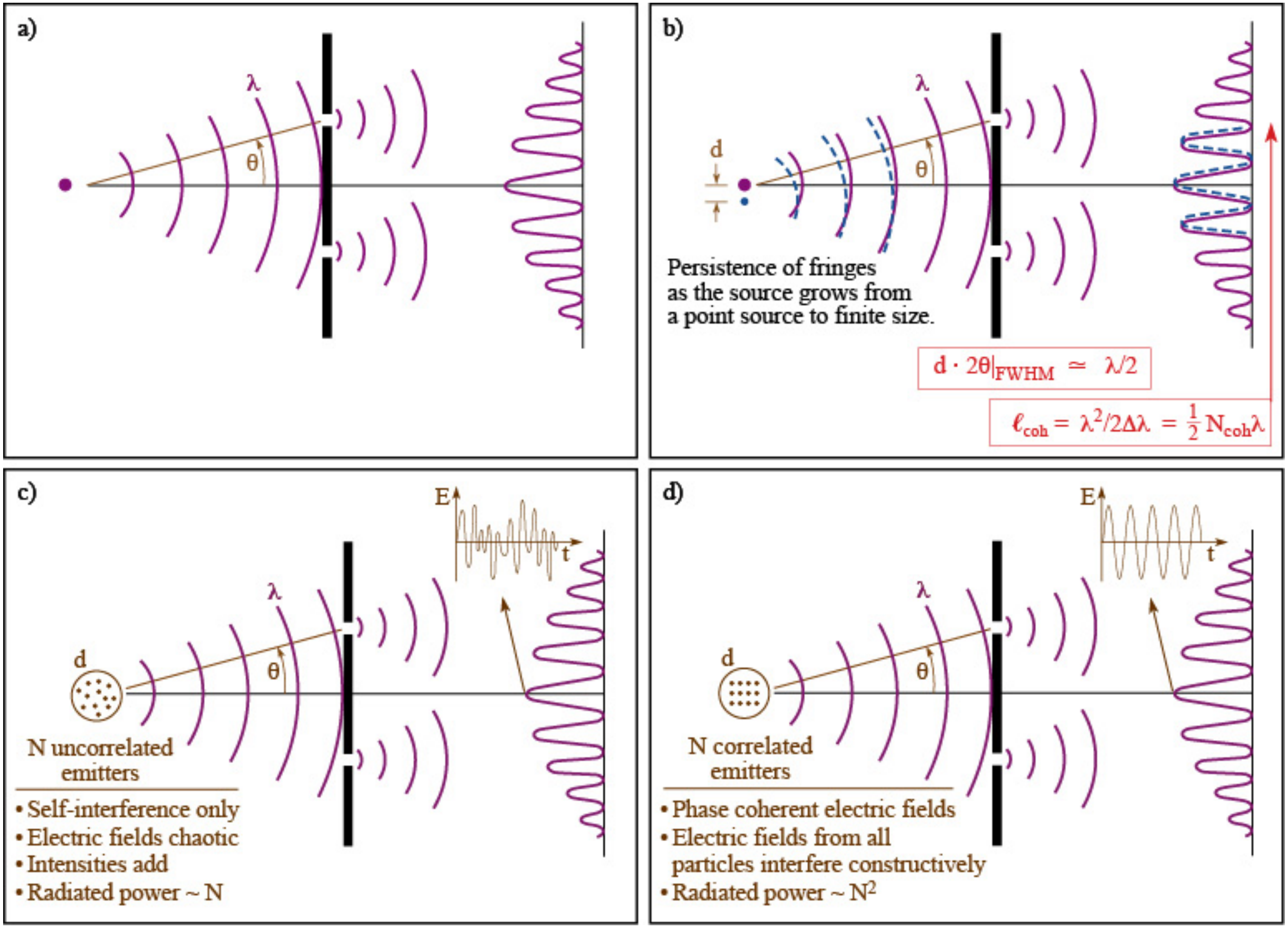
8 GeV, 100 mA
λ_u = 32 mm, N = 140
0 ≤ K ≤ 2.46
σ_x = 393 μm, σ_x' = 15.7 μr
σ_y = 4.98 μm, σ_y' = 1.24 μr
θ_{cen} = 6.6 μr
η = 10%



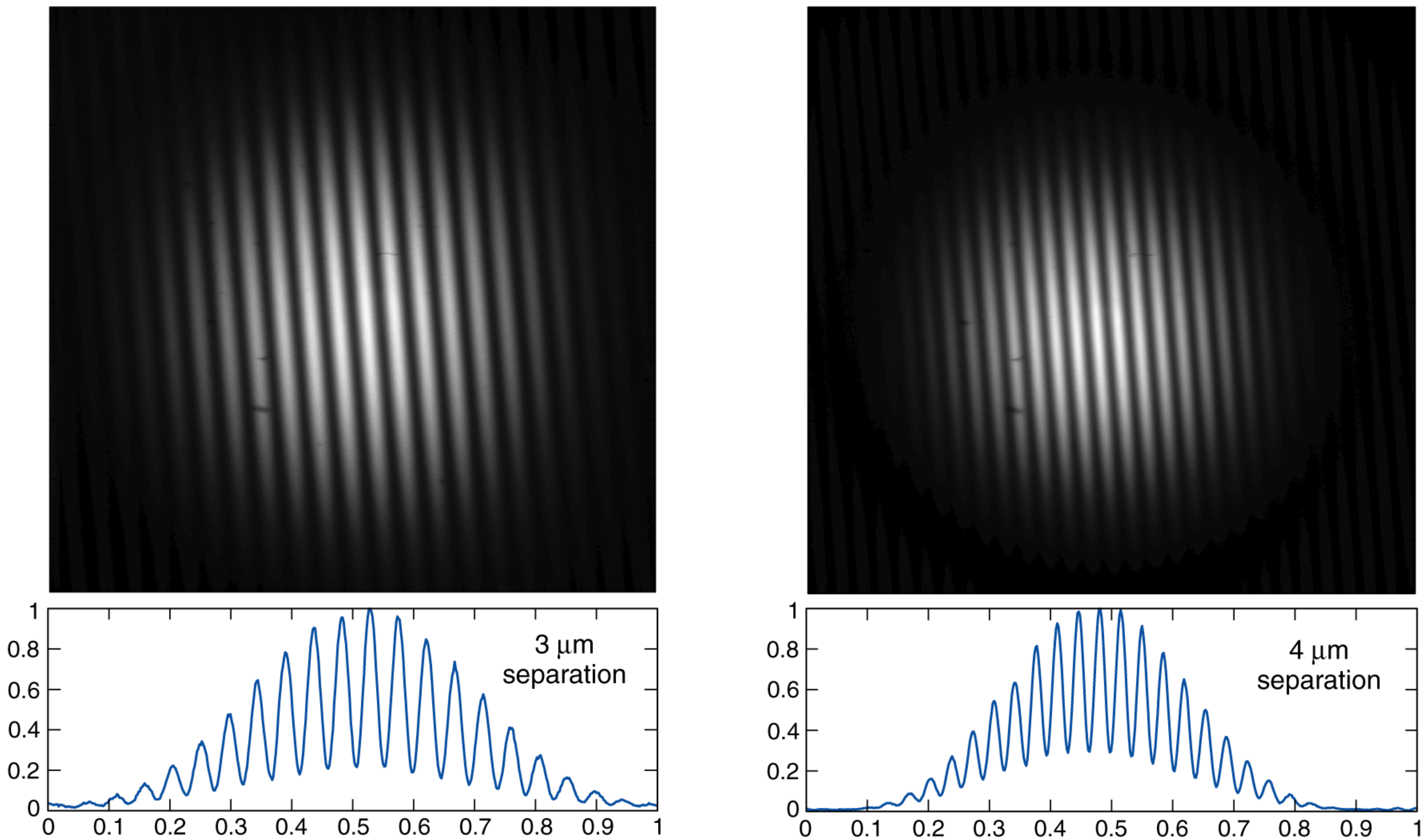


Ch08_SPring8_June2014.ai

Measuring spatial coherence with Young's double slit interferometer



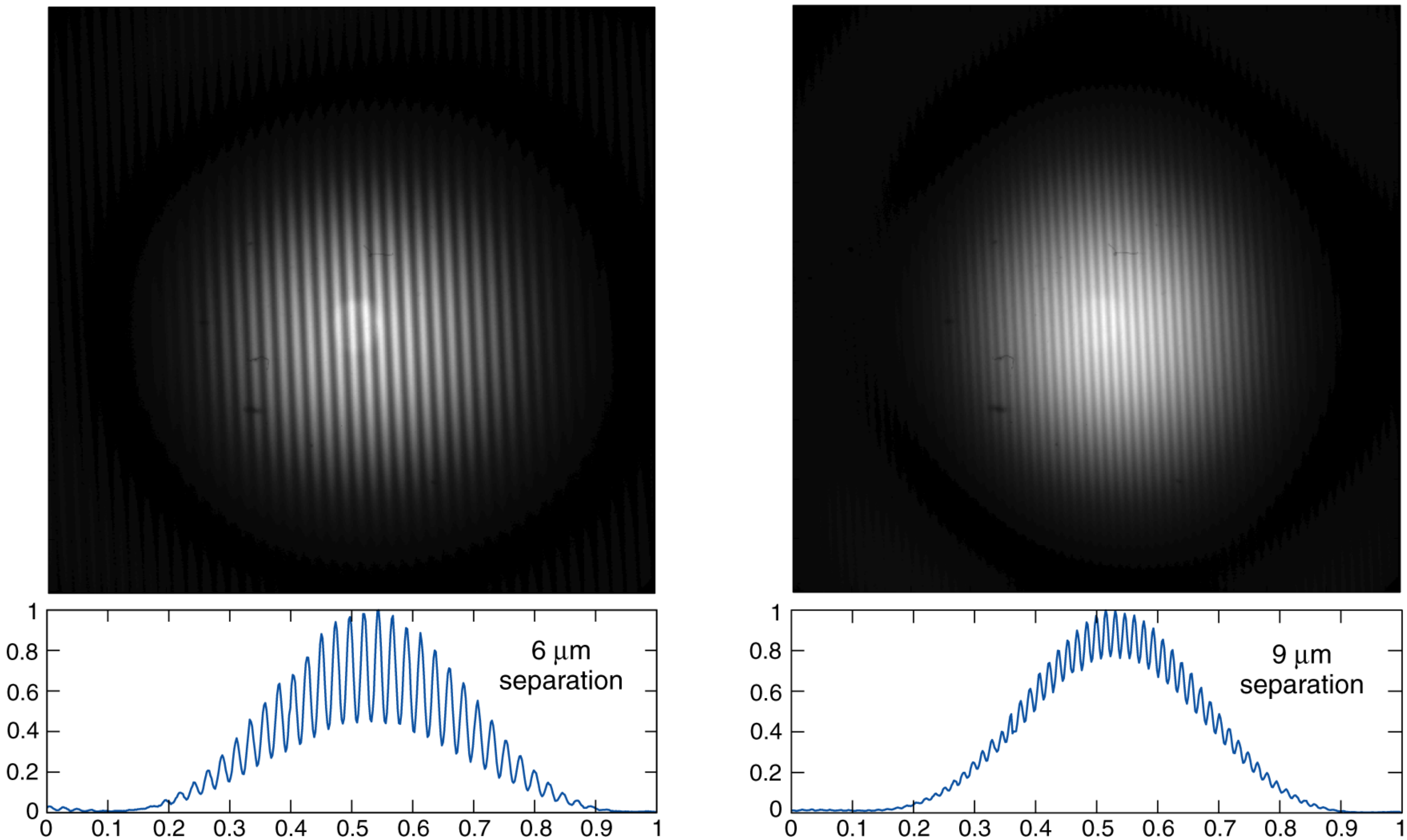
Spatial coherence measurements of undulator radiation using Young's 2-pinhole technique



Courtesy of Chang Chang, UC Berkeley and LBNL.

λ = 13.4 nm, 450 nm diameter pinholes, 1024 x 1024 EUV/CCD at 26 cm ALS, 1.9 GeV, λ_u = 8 cm, N = 55

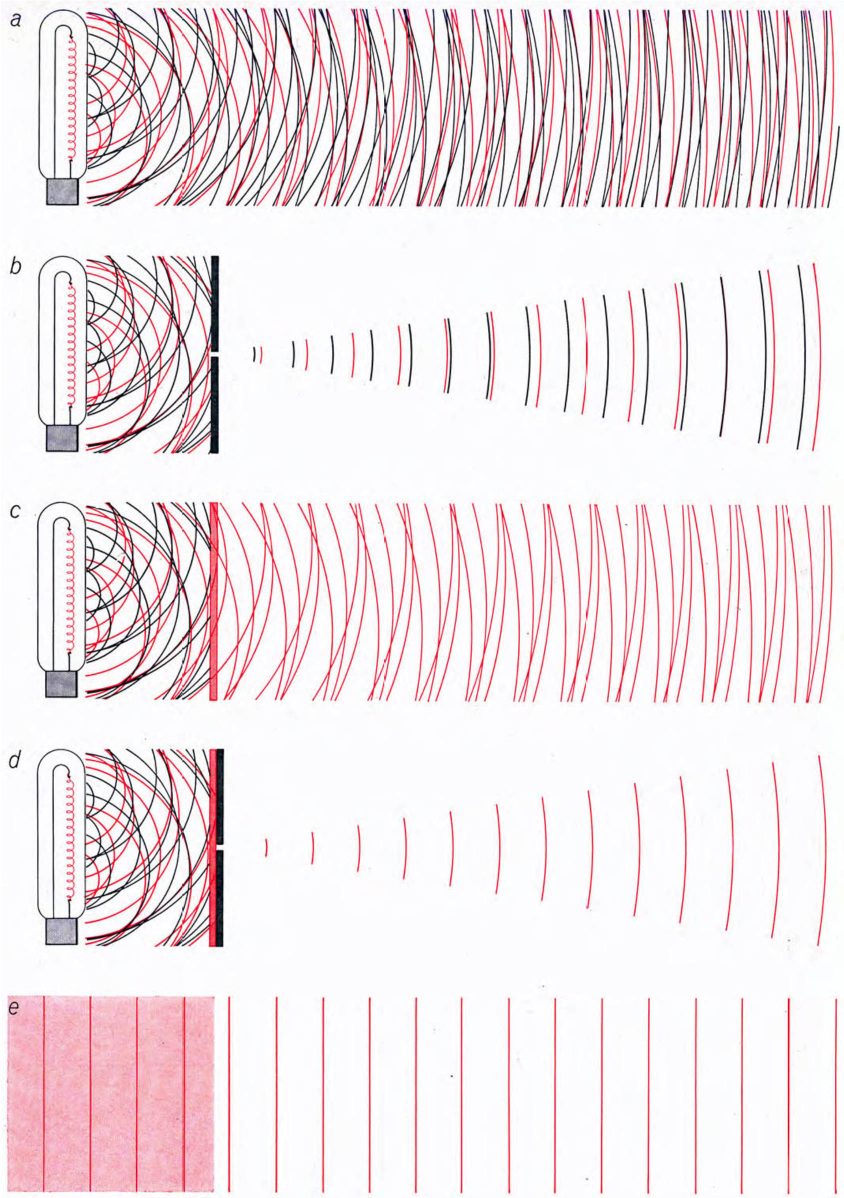
Spatial coherence measurements of undulator radiation using Young's 2-pinhole technique



Courtesy of Chang Chang, UC Berkeley and LBNL.

λ = 13.4 nm, 450 nm diameter pinholes, 1024 x 1024 EUV/CCD at 26 cm ALS, 1.9 GeV, λ_u = 8 cm, N = 55

Ordinary light and laser light



Ordinary thermal light source, atoms radiate independently.

A pinhole can be used to obtain spatially coherent light, but at a great loss of power.

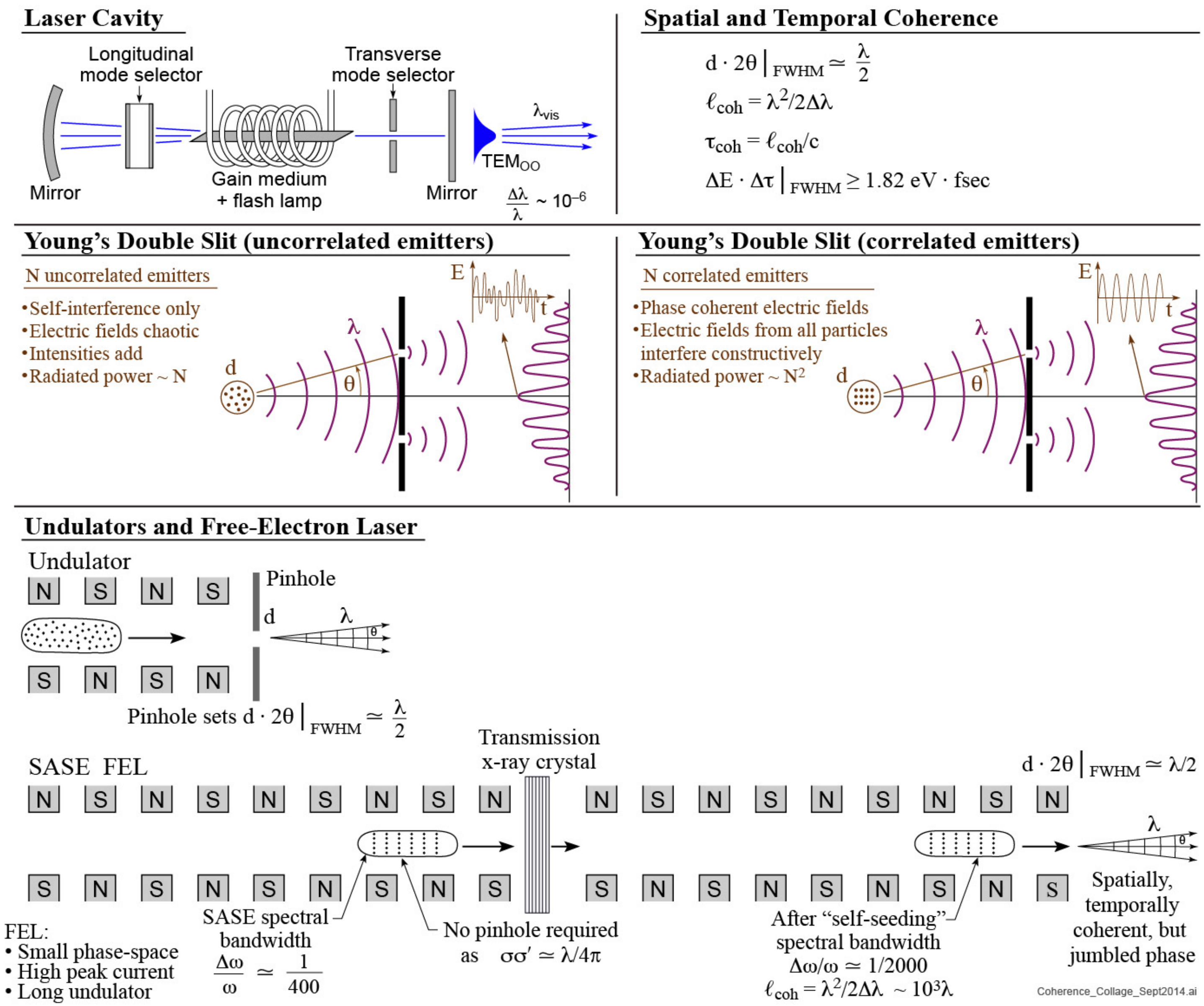
A color filter (or monochromator) can be used to obtain temporally coherent light, also at a great loss of power.

Pinhole and spectral filtering can be used to obtain light which is both spatially and temporally coherent but the power will be very small (tiny).

All of the laser light is both spatially and temporally coherent*.

Arthur Schawlow, “Laser Light”, Sci. Amer. 219, 120 (Sept. 1968)

Coherence: laser, undulator, free electron laser



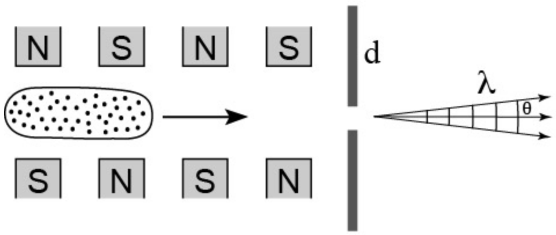
Undulators, FELs and coherence

- Spatial coherence
- Temporal coherence
- Partial coherence
- Full coherence
- Spatial filtering
- Uncorrelated emitters
- Correlated emitters
- True phase coherence and mode control
- Lasers, amplified spontaneous emission (ASE) and mode control
- Undulator radiation
- SASE FEL 10-70 fsec EUV/x-rays
- Seeded FEL true phase coherent x-rays
- High harmonic generation (HHG) compact fsec/asec EUV
- EUV lasers and laser seeded HHG
- Applications with uncorrelated emitters
- Applications with correlated emitters

UndulatorsFELsCoh_Sept2013.ai

Spatial and temporal coherence of undulators and FELs

Undulator

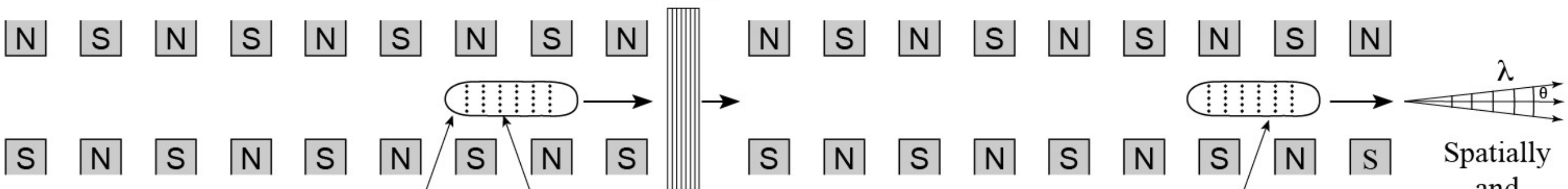


$\frac{\Delta\omega}{\omega} \simeq \frac{1}{N}$ (Noise)

Pinhole sets $d \cdot 2\theta\big|_{FWHM} \simeq \frac{\lambda}{2}$

SASE FEL

Self-seeding transmission x-ray crystal



SASE spectral bandwidth $\frac{\Delta\omega}{\omega} \simeq \frac{1}{400}$

No pinhole required as $\sigma\sigma' \simeq \frac{\lambda}{4\pi}$

$(d \cdot 2\theta\big|_{FWHM} \simeq \frac{\lambda}{2})$

After "self-seeding" spectral bandwidth $\frac{\Delta\omega}{\omega} \simeq 1/2000$

$\ell_{coh} = \frac{\lambda^2}{2\Delta\lambda} \sim 10^3\lambda$

Spatially and temporally coherent, but jumbled phase

FEL:
- Small phase-space
- High peak current
- Long undulator

Future possibilities:
- Is a phase-coherent seed possible at x-ray wavelengths?
- Perhaps a seed plus HGHG multiplication, as at FERMI?
- Is an x-ray FEL oscillator possible?

SpaTemporal_Coh_Undul_FEL_Aug2014.ai

The evolution of incoherent clapping to coherent clapping

The bunching advantage of FELs

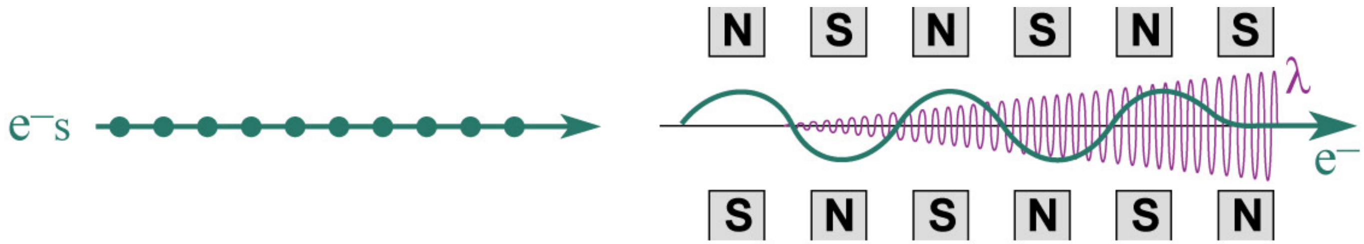
In an undulator with random, uncorrelated electron positions within the bunch, only the radiated self-fields **E** add constructively.

- Coherence is somewhat limited
- Power radiated is proportional to N_e (total # electrons)

For FEL lasing the radiated fields are strong enough to form "microbunches" within which the electron positions are well correlated. Radiated fields from these correlated electrons are in phase. The net electric field scales with N_{ej}, the # of electrons in the microbunch, and power scales with N_{ej}^2 times the number of microbunches, n_j.

- Essentially full spatial coherence
- Power radiated is proportional to $\Sigma n_j N_{ej}^2$; Gain ~ 3×10^6

FEL Physics



- Uniformly distributed particles (beam) into undulator.
- Emission of radiation ("spontaneous" emission).
- Wave grows enough (undulator radiation) to begin affecting. particle dynamics through m**a** = –e**E** radiation.
- Transverse coupling between $\mathbf{E}_{rad}$ and transverse velocity $\mathbf{v}_x$ (in undulator) leads to energy exchange between fields and particle (zero net at first) $\frac{dE_e}{dt} = mc^2 \frac{d\gamma}{dt} = \mathbf{F} \cdot \mathbf{v} = -e\, E \cdot \mathbf{v}_x$
- Modulated velocities with increments in $\mathbf{v}_x$ lead to bunching on axis.
- Electron density modulation leads to stronger radiation,

$$P_{rad} \propto q^2|a|^2 \sim \left(eN_e\right)^2 \cdot \frac{e^2}{m^2} = N_e{}^2 \frac{e^4}{m^2}$$

- Stronger fields (wave) drive stronger transverse velocity.
- Stronger $\mathbf{v}_x$ drives stronger bunching, . . . stronger fields, . . . FEL action.

FEL_physics_Aug2012.ai

Equations of motion for the stronger electric field FEL

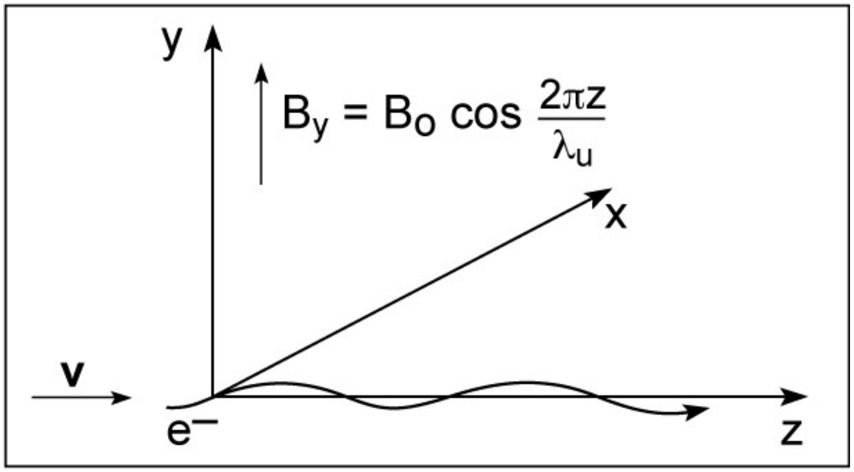
Magnetic fields in the periodic undulator cause the electrons to oscillate and thus radiate. In the FEL case the undulator is very long, the radiated electric field is now strong enough that it can not be ignored in the momentum equation, and the energy of the electrons is no longer constant.

Equations of motion:

$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

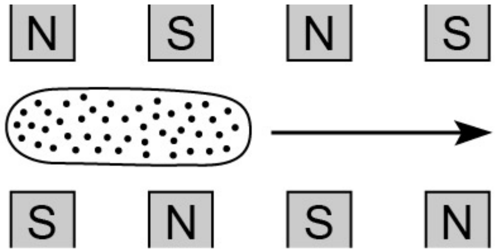
$$\frac{dE_e}{dt} = \mathbf{v} \cdot \mathbf{F} = -e\mathbf{v} \cdot \mathbf{E} = -\frac{eKE}{2mc\gamma}\sin\phi_s$$

where $\mathbf{p} = \gamma m\mathbf{v}$ and $E_e = m\gamma c^2$.



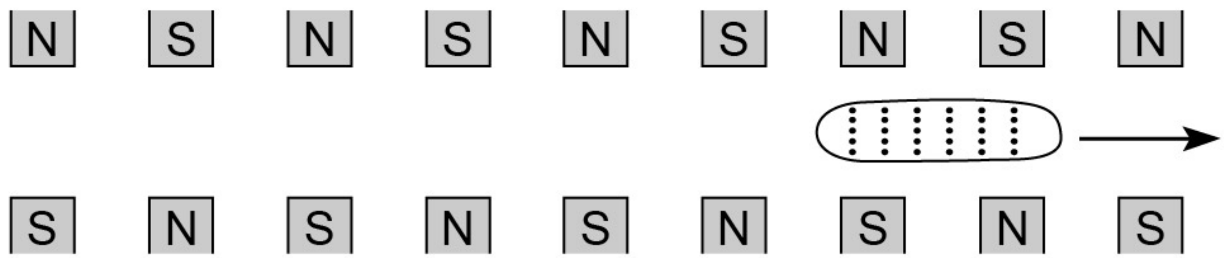
Ch05_F15_Eq16_19.top_Nov2011.ai

Undulators and FELs



$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Undulator – uncorrelated electron positions, radiated fields uncorrelated, intensities add, limited coherence, power ~ N.

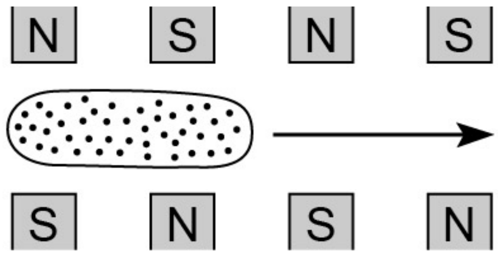


Free Electron Laser (FEL) – very long undulator, electrons are "microbunched" by their own radiated fields into strongly correlated waves of electrons, all radiated electric fields now add, spatially coherent, power ~ N^2

$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

UndulatorsAndFELs2_June2014.ai

Seeded FEL



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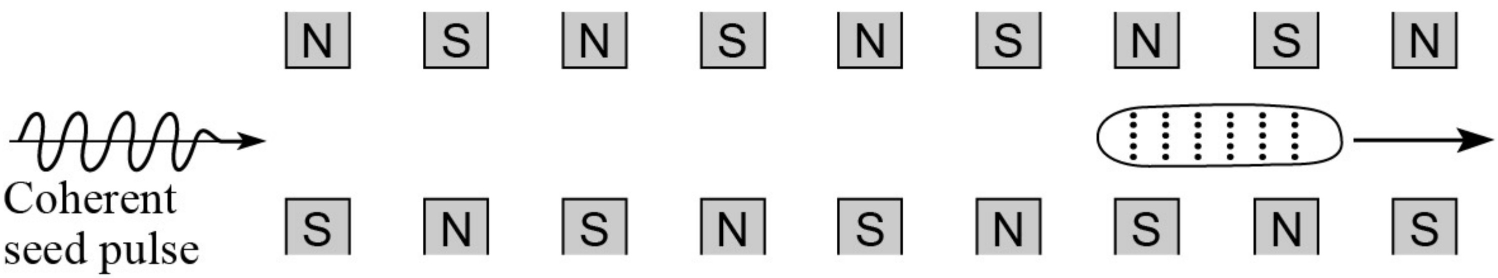

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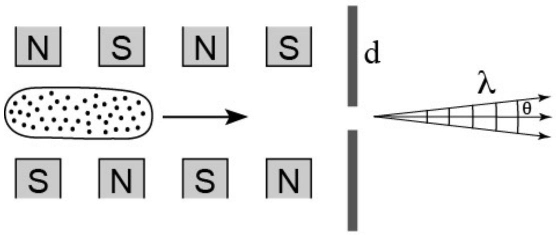
$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Seeded FEL. Initial bunching driven by phase coherent seed laser pulse. Improved pulse structure and spectrum.

UndulatorsAndFELs3_June2014.ai

Spatial and temporal coherence with undulators and FELs

Undulator



$\frac{\Delta\omega}{\omega} \simeq \frac{1}{N}$ (Noise)

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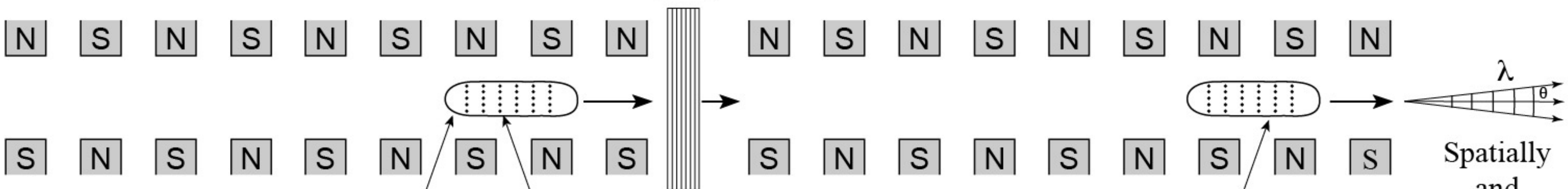
SASE FEL

Self-seeding transmission x-ray crystal



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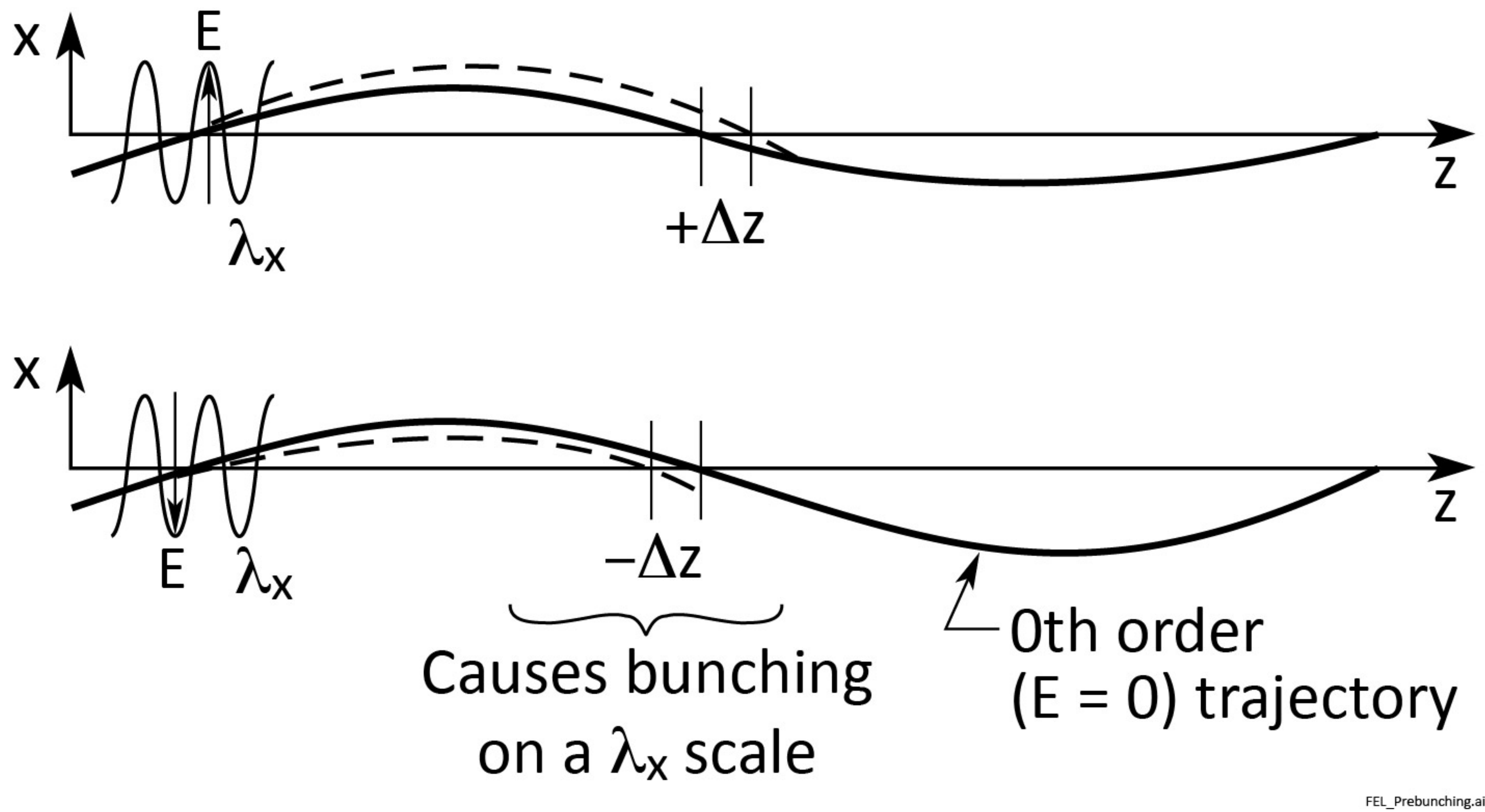
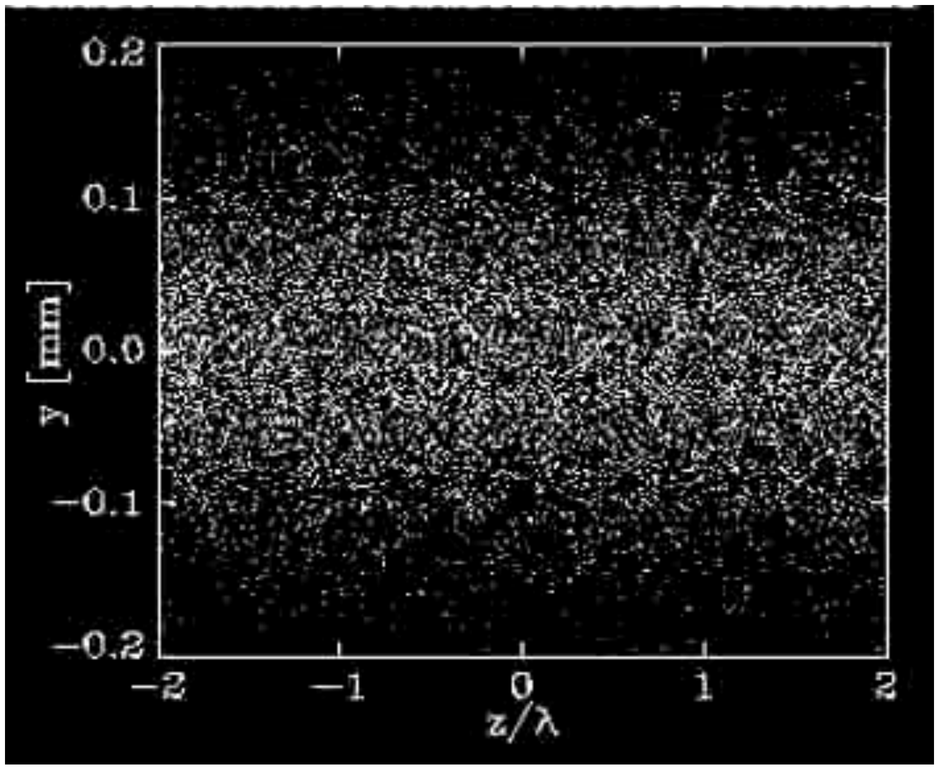
SpaTemporal_Coh_Undul_FEL_Aug2014.ai

Electron energies and subsequent axis crossings are affected by the amplitude and relative phase of the co-propagating field

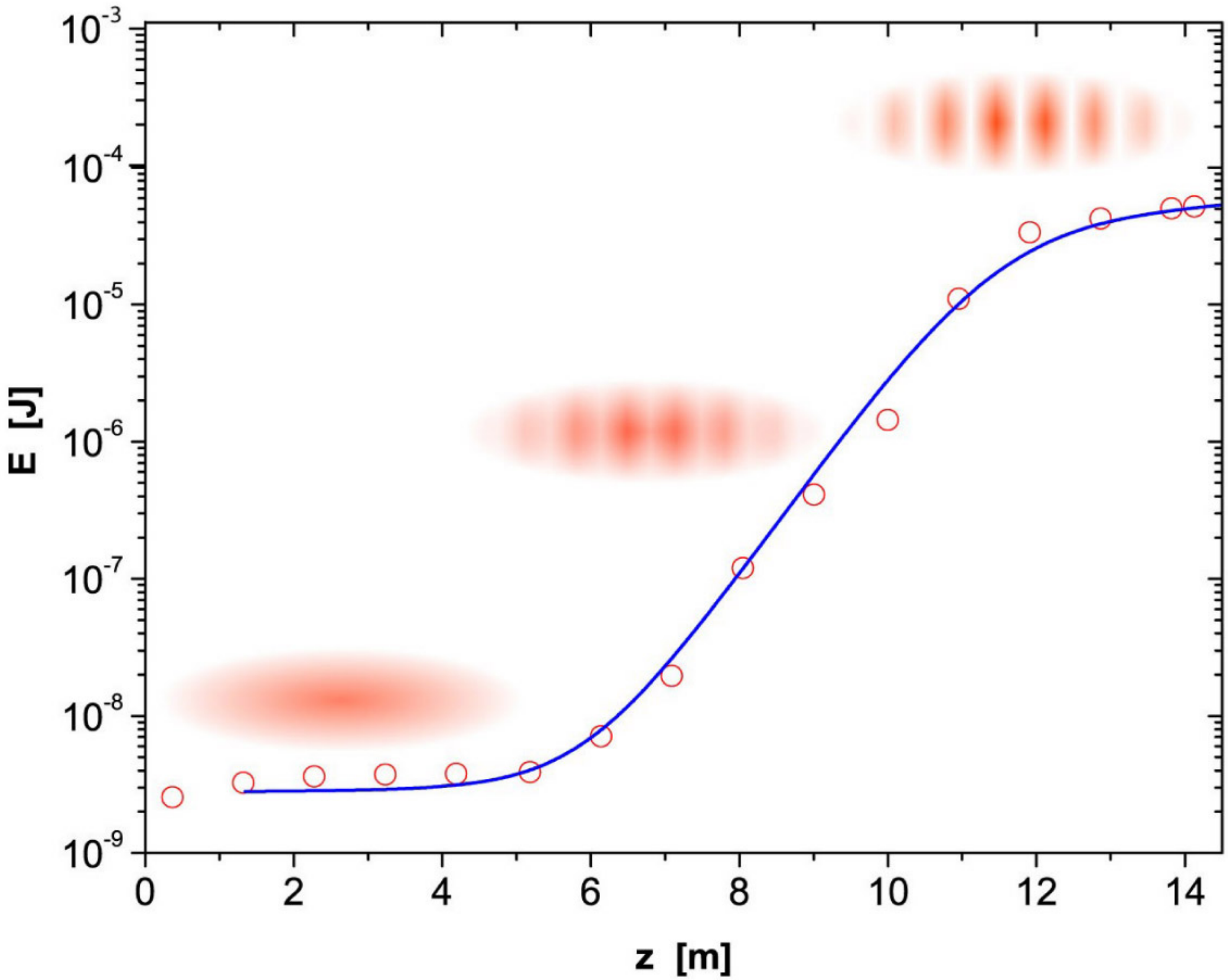


FEL Microbunching



Courtesy of Sven Reiche, UCLA, now SLS

Gain and saturation in an FEL



Courtesy of K-J. Kim

Gain_Saturation_FEL_graph.ai

FEL lasing and the parameter ρ_{FEL}

Equations of motion:

$$\frac{d\mathbf{p}}{dt} = \frac{d}{dt}(\gamma m\mathbf{v}) = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\frac{dE_e}{dt} = mc^2\frac{d\gamma}{dt} = \mathbf{v} \cdot \mathbf{F} = -e\mathbf{v} \cdot \mathbf{E} = -\frac{e\hat{K}E}{2mc\gamma}\sin\phi_s \quad ; \; \hat{K} = K\,[JJ]$$

Strength of wave-particle coupling:

$$\rho_{FEL} = \left(\frac{\hat{K}^2\, r_e n_e \lambda_u{}^2}{32\pi\gamma^3}\right)^{1/3}$$

Lasing gain length (power e-folding):

$$L_{Gain} = \lambda_u/4\pi\sqrt{3}\,\rho_{FEL}$$

Saturated lasing power:

$$\hat{P}_{sat} \simeq \rho_{FEL} \cdot \gamma mc^2 \cdot \hat{I}/e$$

Limited electron energy spread for efficient energy transfer:

$$\sigma_\gamma/\gamma < \rho_{FEL}$$

Electron angular limit for constructive interference:

$$\sigma' \leq 1/\gamma^*\sqrt{N}$$

Electron phase-space ("emittance") match to coherent radiation phase-space:

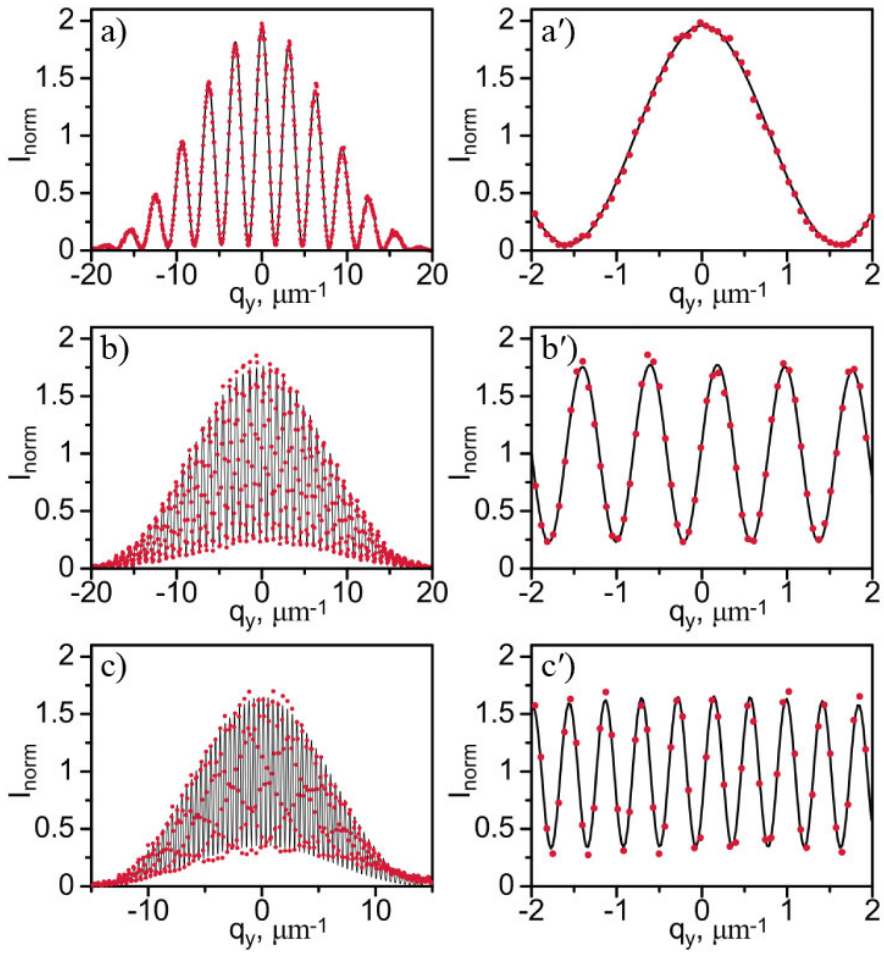
$$\sigma\sigma' \leq \lambda/4\pi$$

Relative spectral bandwidth:

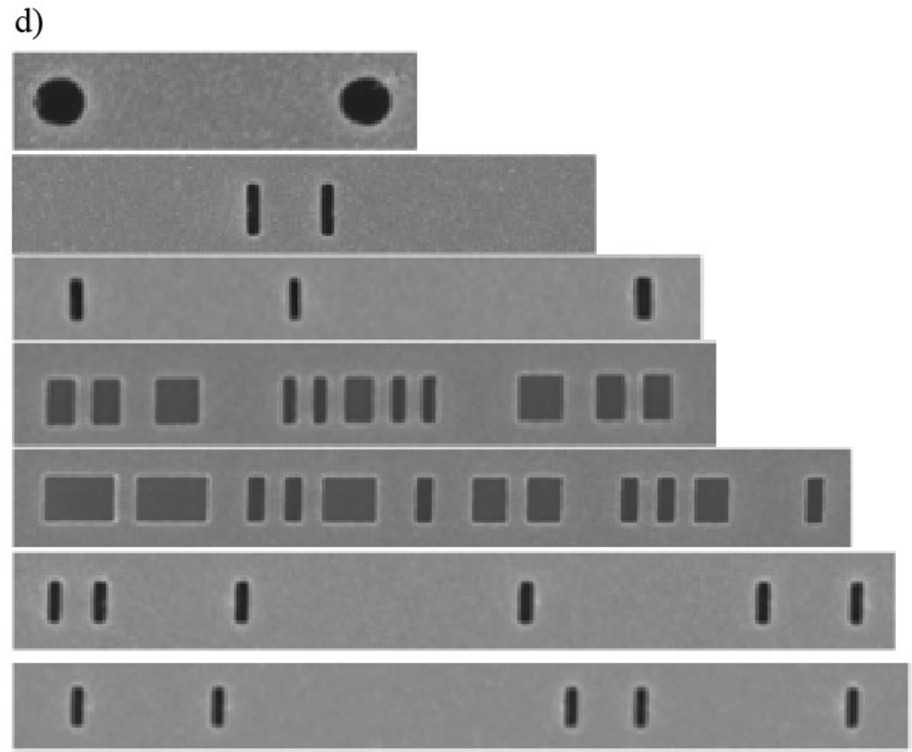
$$\Delta\lambda/\lambda \sim \rho_{FEL}$$

FEL_Lasing_Aug2014.ai

Measuring FEL spatial coherence



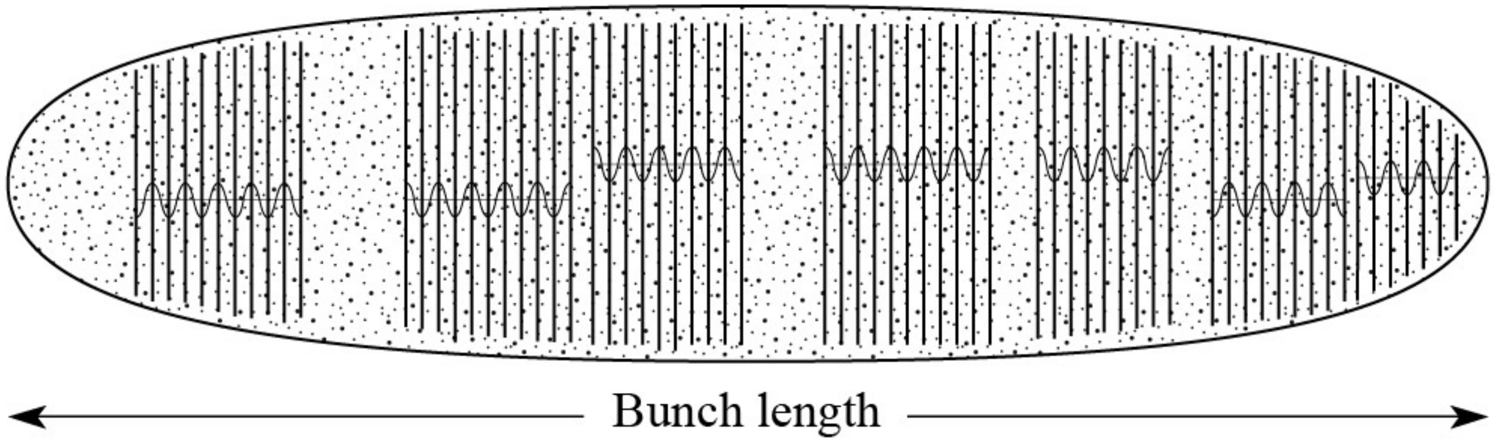
LCLS, 780 eV, 300 fsec, ¼ nC,1mJ/pulse
78% energy in TEM_{00} mode



FEL_Ch6_F12_forSlides.ai

Courtesy of I. Vartanyants (DESY) and A. Sakdinawat (SLAC); PRL **107**, 144801 (30Sept2011)

The coherence properties of Free Electron Lasers



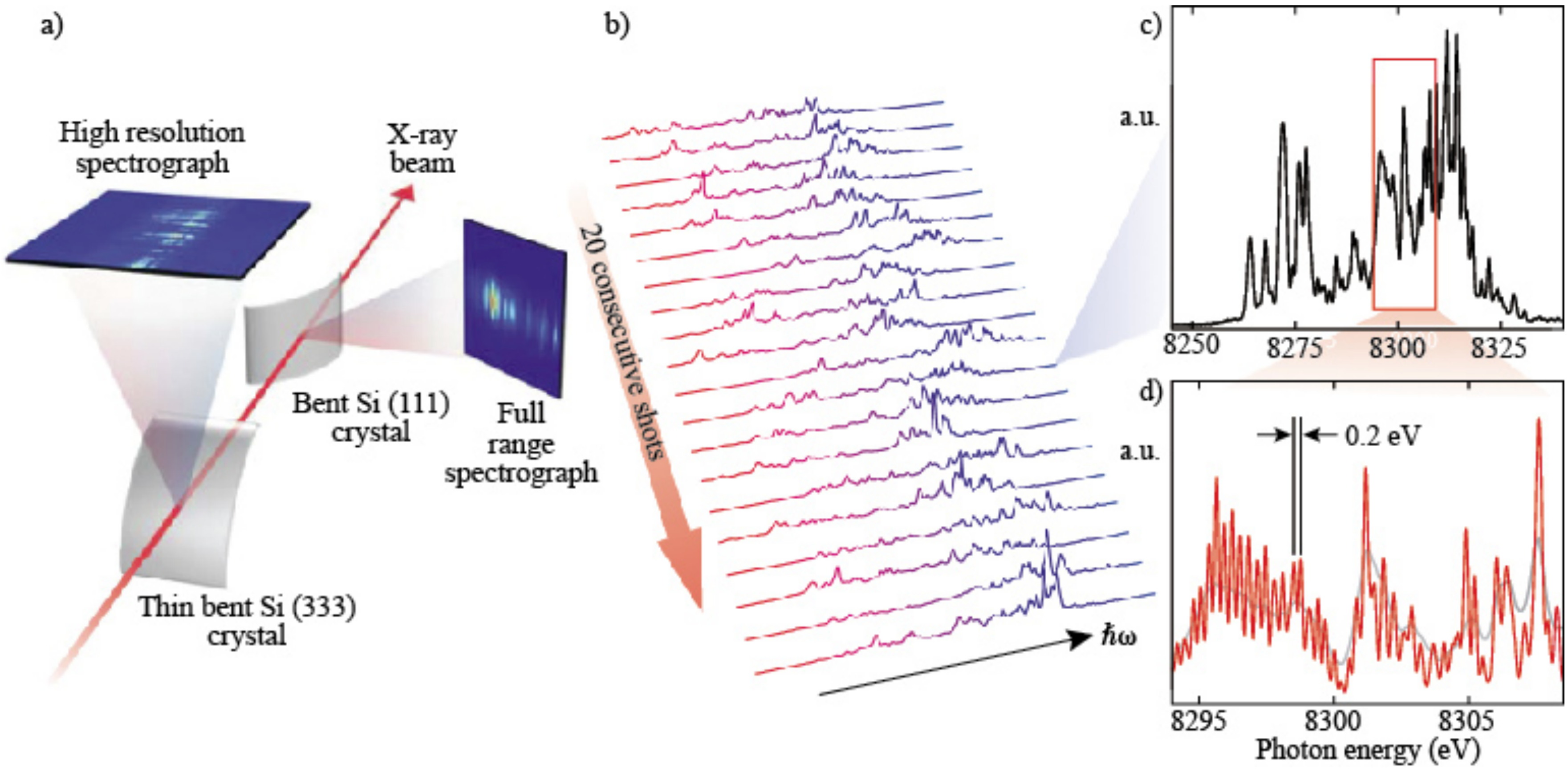
While "SASE" FEL radiation is to a large extent spatially coherent, the electron bunch length can support many independent coherent micro-bunches, thus temporal coherence is limited and spectral bandwidth relatively broad. This is a natural property of the "Self Amplified Spontaneous Emission" (SASE) FEL process. Coherent seeding with a sufficiently intense laser pulse seeks to overcome this limitation, with future plans largely centered on high harmonics of visible/IR lasers of higher power than presently available. Filtered "self-seeding" is also under consideration.

(LCLS: $2.35\,\sigma_Z = 21\ \mu\text{m}$ FWHM vs. $\sim 400 \times .15\ \text{nm} \simeq 0.06\ \mu\text{m}$)

($\therefore$ Can support 300+ microbunches, depending on bunch/pulse duration)

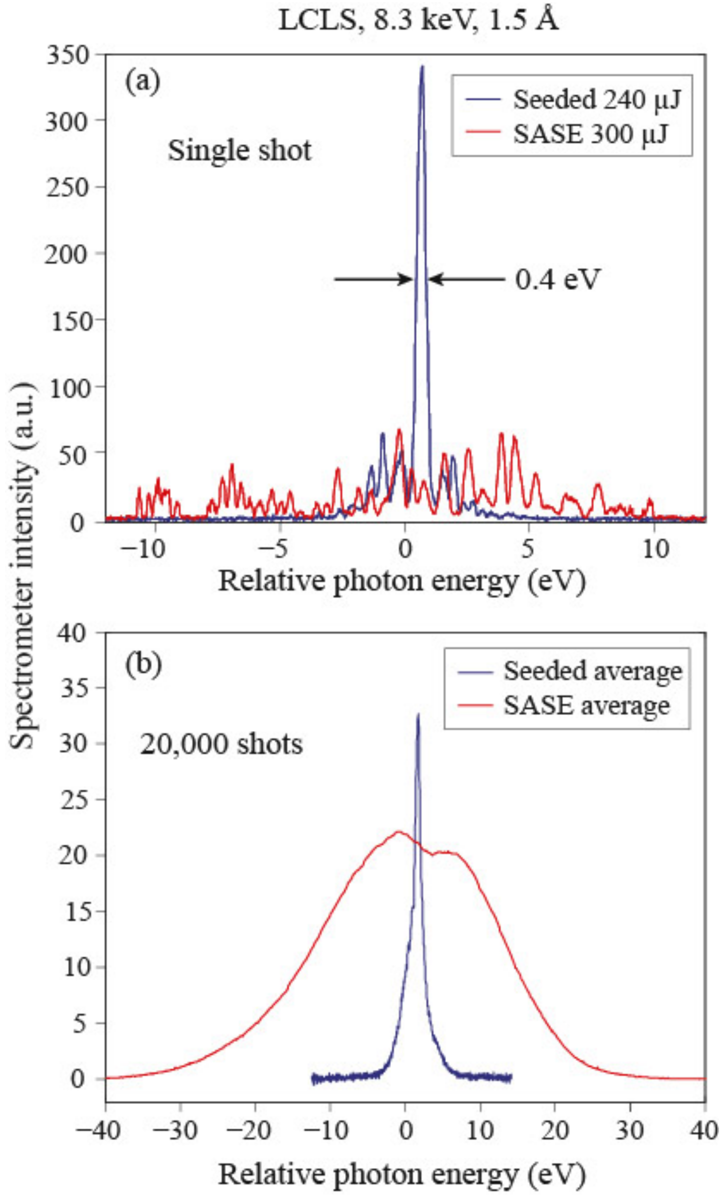
CohPropertiesFEL_6Sept2014.ai

Measuring the FEL SASE spectrum



Courtesy of D. Zhu, J. Hastings, W. Feng, (SLAC); APL **101**, 034103 (July 2012)

Reduced FEL SASE spectra, thus longer coherence length, with “self-seeding”



$$\frac{20\text{ eV}}{0.4\text{ eV}} = 50{:}1\text{ reduction}$$

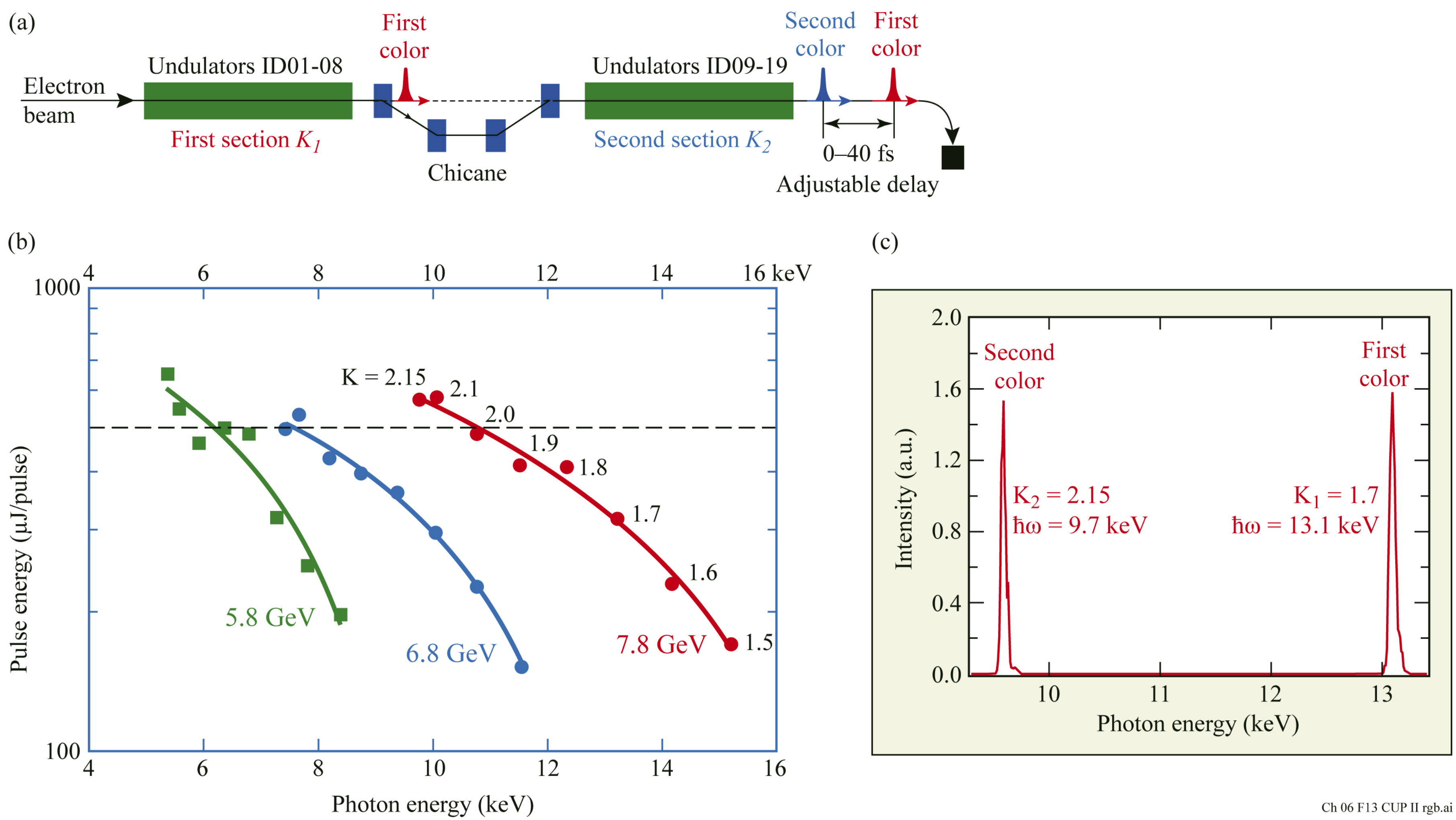
$$\frac{\Delta\lambda}{\lambda} = \frac{8\text{ keV}}{0.4\text{ eV}} = 2\times10^4$$

$$\ell_{coh} = \frac{\lambda^2}{2\,\Delta\lambda} = \frac{2\times10^4}{2}\times 1.5\text{ Å}$$

$$\ell_{coh} = 1.5\ \mu\text{m}$$

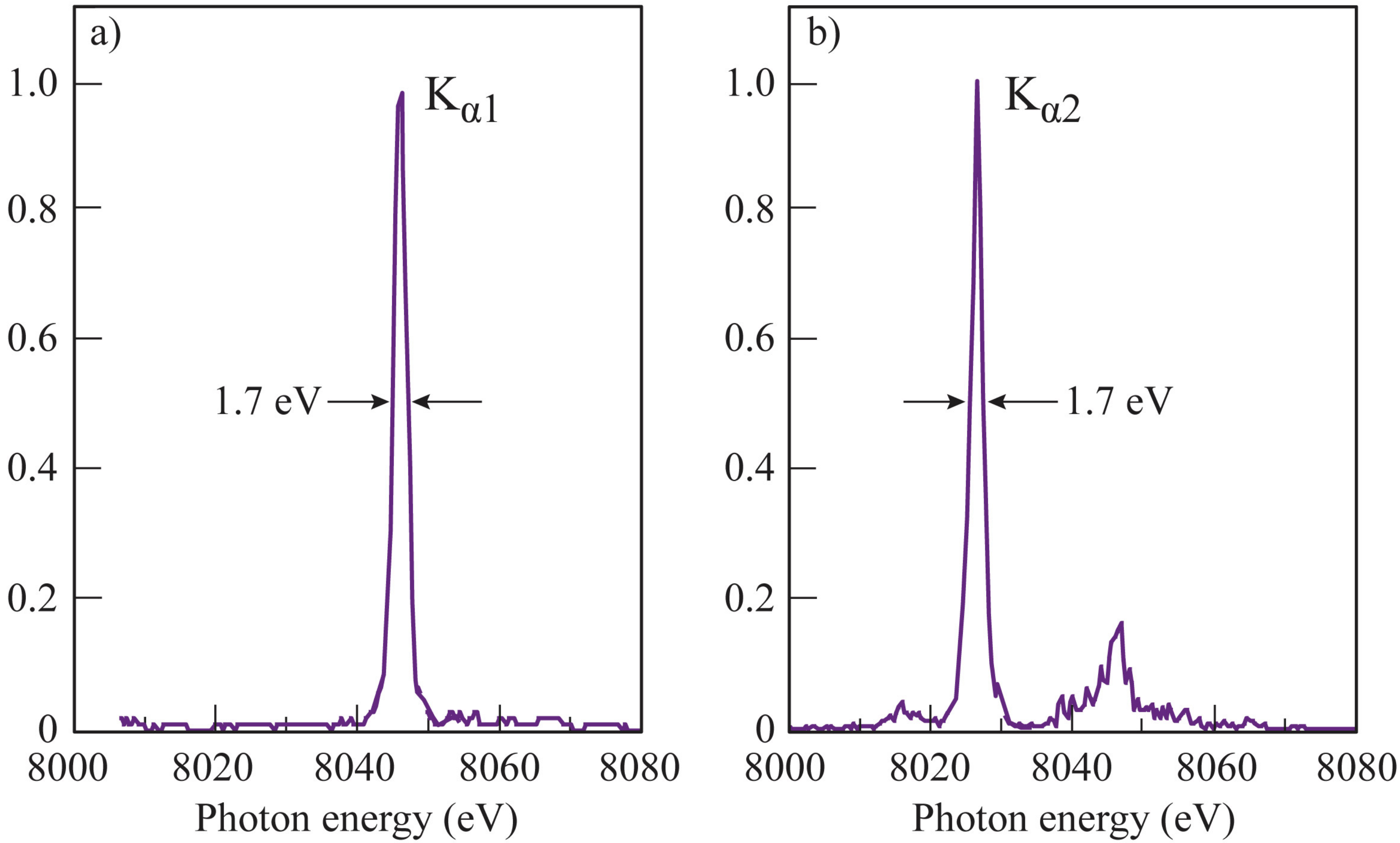
J. Amann, et al.,
Nature Photonics **6**, 693 (Oct 2012)

Pump-probe capabilities at SACLA



Courtesy of H. Tanaka, SACLA

FEL pumped, narrow line, Cu K_α atomic laser at 1.54 Å



Ch 06 F14 CUP II rgb.ai

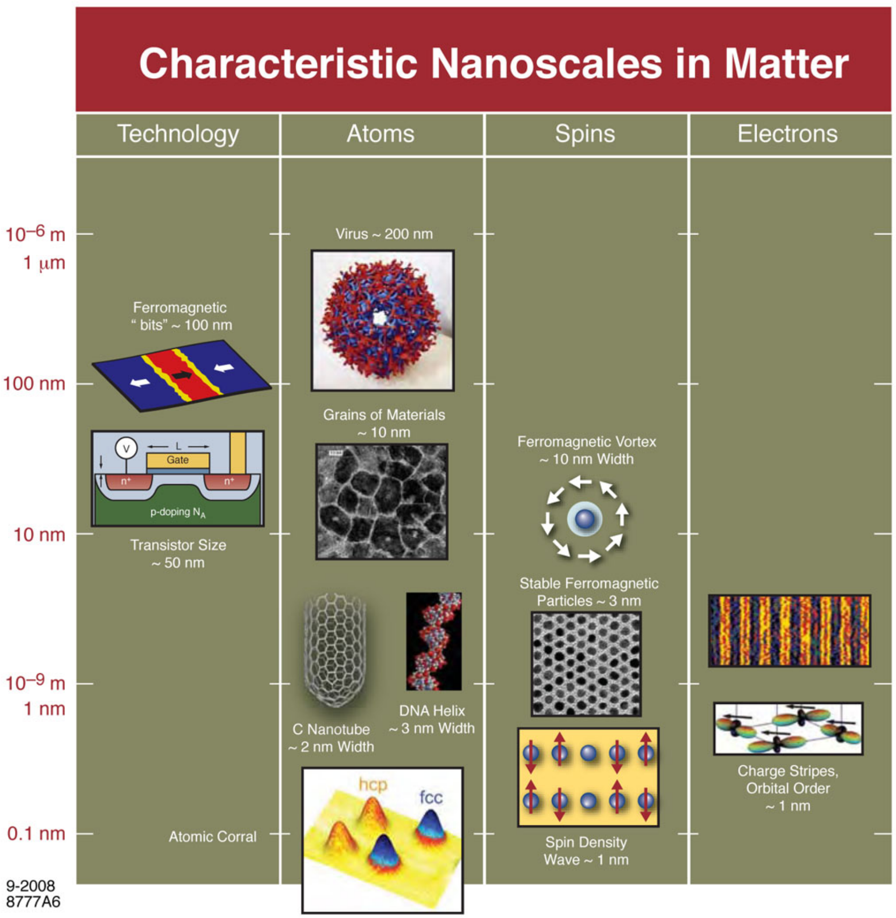
Courtesy of M. Yabashin, SACLA

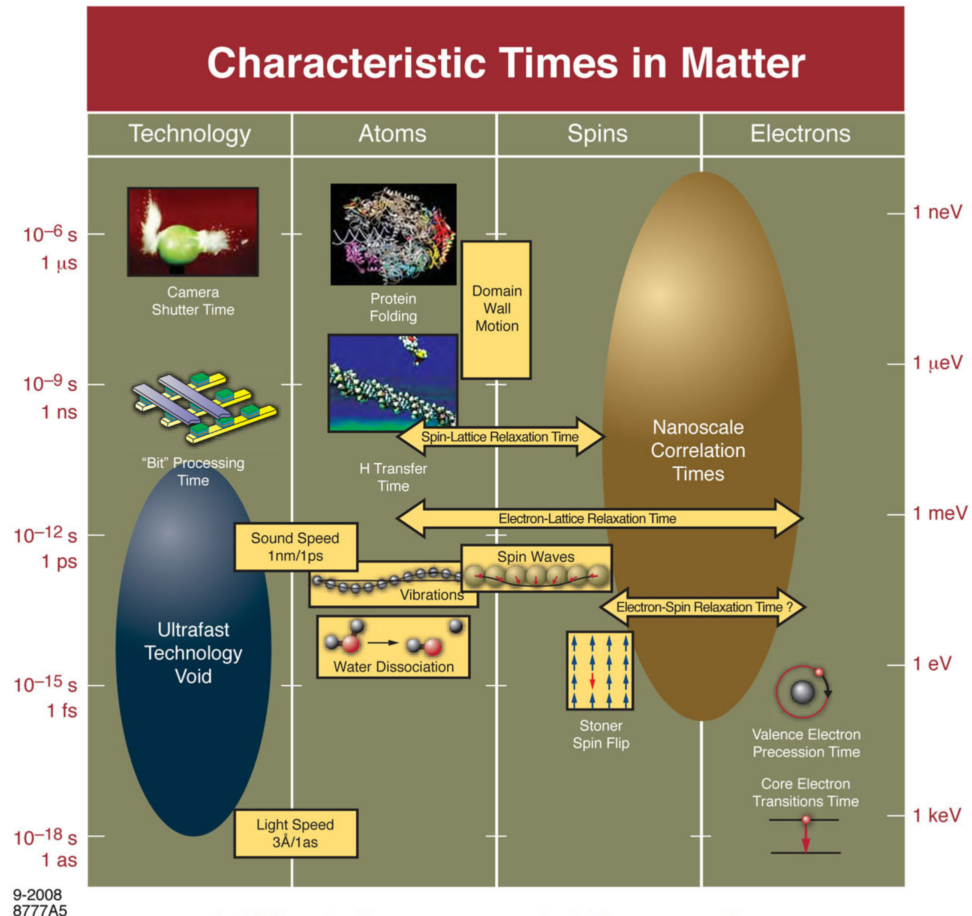
Typical FEL parameters

Parameters	FLASH FEL (Hamburg 2005)	Fermi (Trieste, 2010)	LCLS (Stanford, 2009)	SACLA (Hyogo, 2011)	Swiss FEL (PSI, 2016)	PAL (Pohang, 2016)	EU XFEL (Hamburg, 2016)
E_e	1.25 GeV	1.5 GeV	13.6 GeV	8 GeV	5.8 GeV	10 GeV	17.5 GeV
γ	2,450	2,900	26,600	15,700	11,300	19,600	35,000
$\hat{I}$	1.3 kA	300 A	3.4 kA	9 kA	3 kA	3 kA	5 kA
λ_u	27.3 mm	55 mm	30 mm	18 mm	15 mm	26 mm	40 mm
N	989	216	3733	4986	3200	3456	4375
L_u	27 m	14 m	112 m	90 m	48 m	90 m	175 m
$\hbar\omega$	30-300 eV (4-40 nm)	20-300 eV (4-60 nm)	250 eV-10 keV (1.2-50 Å)	4.5-15 keV (0.8-2.8 Å)	2-12 keV (1-6 Å)	2-12 keV (1-6Å)	3-25 keV (0.05-4Å)
$\lambda/\Delta\lambda_{FWHM}$	100	1000	200-500	200-400	200-500	200-500	1000
$\Delta\tau_{FWHM}$	50 fsec	85 fsec	<10-70 fsec	<10 fsec	2-20 fsec	5-60 fsec	100 fsec
$\dot{\mathcal{F}}$ (ph/pulse)	3×10^{12}	5×10^{12}	2×10^{12}	4×10^{11}	10^{11}	5×10^{11}	10^{12}
rep rate	1 MHz @ 10 Hz	10 Hz	120 Hz	60 Hz	100 Hz	60 Hz	2.7 kHz @ 5 Hz
$\hat{P}$	1 GW	1 GW	30-60 GW	60 GW	3 GW	15 GW	20 GW
L	260 m	200 m	2 km	710 m	730 m	1.1 km	3.4 km
Polarization	linear	variable	linear	linear	Linear	Linear	Linear
Mode	SASE (3ω Ti: saphire)	Seeded	SASE & self-seeded	SASE	SASE & self-seeded	SASE & self-seeded	SASE

FreeElectronLasersChart_July2015.ai

Probing matter on the scale of nanometers and femtoseconds

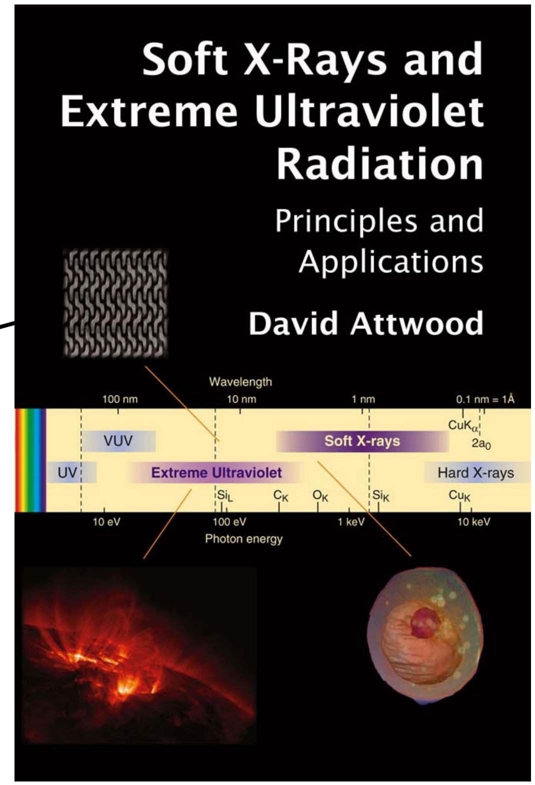




$$(\Delta E \cdot \Delta\tau)_{FWHM} = 1.82\ \text{eV} \cdot \text{fsec}$$

Science and Technology of Future Light Sources (Argonne, Brookhaven, LBNL and SLAC: Four lab report to DOE/Office of Science, Dec. 2008)

2^{nd} Edition in progress: new FEL, HHG, Coherence, and X-ray Imaging chapters



UC Berkeley

www.coe.AST.berkeley.edu/sxr2009
www.coe.AST.berkeley.edu/srms
www.youtube.com